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PHOTON SHADOWING IN NUCLEI

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ملخـــــــص

تمت دراسة ظلال الفوتونات في الانوية باستخدام طريقة جلوبر ـ فيكتور ميزون المعممة . اجريت الدراسة باستخدام الطريقتين القطرية وغير القطرية لـ فيكتور ـ ميزون المعممة ـ وكانت النتائج النظرية متفقة مع النتائج العملية مراعين عدم يقينية النتائج العملية المتوفرة .

ABSTRACT

Photonuclear Shadowing is studied using the Glauber - generalized vector meson dominance approach. The calculation is performed using both diagonal and off - diagonal generalized vector dominance (GVMD) models. The results are compatible with experiments within the large systematic uncertainties of the available data.

INTRODUCTION

The most striking feature of photon induced reactions at high energy is their similarity to purely hadronic reactions. The behaviour with energy, with momentum transfer, and the quantum number systematics of both sets of reactions are identical. The only difference is that the cross sections for photon induced reactions are reduced by a factor of $\sim \alpha$ compared to hadronic cross sections[2,14,19].

As the photon propagates through space, it fluctuates back and forth between the bare photon state, some collection of hadronic states, and a set of electromagnetic states. That is to say that the physical photon can spend a fraction α of its time as a superposition of hadrons; i.e

$$|\gamma \text{ physical}\rangle = |\gamma \text{ bare}\rangle + \sqrt{\alpha}\,|\text{hadrons}\rangle + \sqrt{\alpha}\,|e^+e^-\rangle + O(\alpha) \quad (1)$$

So if the photon, while in the midst of hadronic fluctuation, hits a target it will interact as a hadron.

Because hadron processes shadow in nuclei, thus photon processes should also shadow in nuclei. The effect of shadowing is most easily seen for different nuclei in terms of the effective nucleon number, defined as

$$\frac{A_{eff}}{A} = \frac{\sigma_{\gamma A}}{A\,\sigma_{\gamma N}} \qquad (2)$$

Experiments show that real as well as spacelike photons shadow in nuclei at energies above the transition energy[3,8,9,16,17,18,20,24]. given by

$$\phi = \frac{M^2 + Q^2}{2\,n_0\,\sigma_{hN}(M^2)} \qquad (3)$$

where $\sigma_{hN}(M^2)$ is the total cross section for the hadronic system h of mass M upon a nucleon, n_0 is the constant nuclear density, and Q^2 is the mass of the virtual photon.

The aim of this paper is to see if generalized vector dominance (GVMD) models can account for nuclear shadowing data including the photoproduction data of Eisner[10], and in more detail to see if present data can discriminate between different versions of GVMD models. Similarly, to see if future high energy electroproduction data can help in distinguishing between the general classes of vector dominance models.

METHOD

Generalized vector meson dominance (GVMD) model: The failure of simple vector dominance (SVD) model, in which electromagnetic interactions are mediated by the low - lying vector mesons $V = \rho, \omega, \phi$, to account for the photoabsorption cross section upon nucleons, strongly suggests the incorporation in SVD model of all the higher - mass hadronic states created in e^+e^-- annihilation to order $\sqrt{\alpha}$. This leads to GVMD models.

For transverse photons, the general form of the GVMD model, for the spin-averaged, forward compton scattering amplitude is given by

$$F^T_{\gamma\gamma} = \sum_{m,n} \left(\frac{e}{f_m}\right) \frac{m_m^2}{m_m^2 + Q^2} F^T_{mn} \frac{m_n^2}{m_n^2 + Q^2} \left(\frac{e}{f_n}\right) \qquad (4)$$

where F^T_{mn} is the forward vector — meson scattering amplitude for $V_m N \rightarrow V_n N$ and f_m, f_n are the $\gamma - V_{m,n}$ coupling constants which relate $F^T_{\gamma\gamma}$ to F^T_{mn} via vector dominance.

In the diagonal GVMD model only the diagonal (m=n) transitions are retained in the sum of equation (4)[21,22,23]. while in the off-diagonal GVMD nearest off-diagonal (m=n±1) transitions as well as diagonal transitions are retained in the sum.

The contribution of off - diagonal terms was first discussed by Cocho *et. al.*[5] and Fraas *et. al.*[11] who proposed a simple model incorporating them for purely diffractive scattering. Subsequently, Ditsas[7] and Ditsas & Shaw[6] extended Fraas (FRS) model to include non - diffractive terms at the price of a second free parameter, and their extended version is adopted in this calculation. For GVMD models one assumes a Veneziano type spectrum of vector mesons whose masses satisfy

$$m_n^2 = m_0^2 (1 + \lambda n) \quad , \quad n = 0, 1, 2, \ldots \qquad (5)$$

with $m^{\circ} = m_p$ being equal to the mass of the first member of the series and λ is put equal to 2. The photon vector - meson $\gamma - V_{m,n}$ couplings satisfy

$$m_n^2 f_0^2 = m_0^2 f_n^2 \qquad (6)$$

which leads to $^1/S$ behaviour for the high-energy $e^+ e^- \longrightarrow$ hadrons[15], with $f_0^2/4\pi = 2.36$.

In the diagonal calculation the vector - meson nucleon cross sections are taken to be mass-dependent as imposed by scaling in deep inelastic electron scattering,

$$\sigma_{nN}(S) = \frac{m_\rho^2}{m_n^2} \sigma_{\rho N} \qquad (7)$$

where

$$\sigma_{\rho N} = 21.9 + 13.5 \, \nu^{-\frac{1}{2}} \; mb \qquad (8)$$

while in the off–diagonal calculation the vector–meson nucleon cross sections are taken to be mass–independent as imposed by scaling

$$\sigma_{nN}(S) = \sigma_{\rho N} \qquad (9)$$

The transverse-photon nucleon cross section is taken to be

$$\sigma_{\gamma N} = \frac{1}{2} (\sigma_{\gamma p} + \sigma_{\gamma n}) = 77.9 + 58.5 \, \nu^{-\frac{1}{2}} \; \mu b \; (I_\gamma = 1) \qquad (9)$$

Shadowing Calculation: A detailed theoretical model of nuclear shadowing in electromagnetic processes was first done by Brodsky & Pumplin[2] and Margolis & Tang[19], all of whom used the eikonal optical-potential approach[12,13]. In the multichannel case[14], L hadronic systems – I = 1 – are considered to mediate the photon-nucleon interactions. The profile function for V-photoproduction becomes a column vector, and the profile function for the inverse process becomes a row vector. Similarly the profile function for elastic V–nucleus becomes a symmetric matrix

$$T_\gamma = \begin{pmatrix} T_{1\gamma} \\ \vdots \\ T_{L\gamma} \end{pmatrix} \qquad , T_\gamma^t = \left(T_{1\gamma} \cdots\cdots T_{L\gamma} \right) \tag{11}$$

$$T = \begin{pmatrix} T_{11} & \cdots\cdots & T_{1L} \\ & & \\ T_{L1} & & T_{LL} \end{pmatrix}$$

Define the matrices

$$\Lambda = \begin{pmatrix} \lambda_{11} & \cdots & \lambda_{1L} \\ & & \\ \lambda_{L1} & & \lambda_{LL} \end{pmatrix} \tag{12}$$

$$\Delta = \mathrm{diag}(\Delta_1 , \cdots \Delta_L)$$

where the λ's are the inverse mean-free paths and

$$\Delta_n = K_n - K \cong - \frac{m_n^2 + Q^2}{2\nu} \tag{13}$$

is the minimum momentum transfer for photoproducing the state n. In terms of these quantities the matrix generalization of the two - step nucleon compton amplitude[14] is

$$F_{\gamma\gamma}^{(2)}(\Delta, Q^2) = \left(-\frac{iK}{2\pi}\right) \int d^2b\, e^{-i\Delta . b} \int_{-Z_0}^{Z_0} dZ \int_{-Z_0}^{Z} dZ' \, K_{\gamma\gamma}^{opt} \tag{14}$$

where (15)

$$K_{\gamma\gamma}^{opt}(\underline{b}, Z ; Z') = \tfrac{1}{2} \lambda_\gamma^T \exp\left[-\left(\tfrac{1}{2}\Lambda - i\Delta\right)(Z - Z') \right] \tfrac{1}{2} \lambda_\gamma$$

Carrying out the Z and X integrations for a spherical nucleus with constant nuclear density and specializing to forward scattering with using the opotical theorem, one finds for the effective nucleon number the well–known result[14]

$$\frac{\sigma_{\gamma A}(\nu,Q^2)}{A\,\sigma_{\gamma N}(\nu,Q^2)} \equiv \frac{A_{eff}}{A}$$

$$= Re\left(1 - \frac{1+\mathcal{E}_c}{Re(\lambda_{\gamma\gamma})}\,\lambda_\gamma^T\left[\frac{1-G\big((\Lambda-2i\Delta)R\big)}{(\Lambda-2i\Delta)} - \frac{\mathcal{E}_c}{(\Lambda-2i\Delta\,\mathcal{E}_c)}\right]\lambda_\gamma\right) \tag{16}$$

Where G is a matrix function of a complex symmetric matrix

$$G(X) = \frac{3}{X^3}\left[(1+X)e^{-X} - 1 + \frac{X^2}{2}\right] \qquad , \tag{17}$$

$\mathcal{E}_c = \frac{L_c}{2L_n}$ with L_c being the constant correlation length taken as 0.3 fm and L_n^{-1} is the inverse mean free - path given by $L_n^{-1} = n_o\,\sigma_{hN}(1-i\eta)$.

The numerical calculation was performed by programming equation (16). We start by specifying the scattering amplitudes. The vector–meson nucleon forward scattering amplitude (nN $\longrightarrow$ nN) is assumed to have the form

$$f_{nn}(k,0) = \frac{iK}{4\pi}\,\sigma_{nN}(1-i\eta) \tag{18}$$

where K is the 3 - momentum of the vector–meson in the laboratory frame, η is the ratio of the real to the imaginary part of the forward scattering amplitude, i.e,

$$\eta = -\frac{Re\,f(nN\rightarrow nN)}{Im\,f(nN\rightarrow nN)} \tag{19}$$

and σ_{nN} is the vecotor--meson nucleon cross--section.

The off--diagonal scattering amplitude f(n,n+1) is determined from the relation[6,7]

$$f_{n,n+1} = -\frac{1}{2}\left(\frac{m_n}{m_{n+1}}\right)\frac{ik}{4\pi}\left[\left(1-2\delta_D\frac{m_0^2}{m_n^2}\right)\sigma_{\rho N}^{D} + \left(1-2\delta_R\frac{m_0}{m_n}\right)\sigma_{\rho N}^{R}\right] + \frac{1}{2}\left(\frac{m_n}{m_{n+1}}\right)\left(1-2\delta_R\frac{m_0}{m_n}\right)\frac{k}{4\pi}\sigma_{\rho N}^{R} \qquad (20)$$

with $\delta_D = 0.15$ and $\delta_R = 0.23$.

Photoproduction amplitudes are related to vector–meson nucleon amplitudes via VMD relation

$$f_{\gamma N \to nN}(\nu, Q^2) = \sum_m \left(\frac{e}{f_m}\right)\frac{m_m^2}{m_m^2+Q^2} f_{mn} \qquad (21)$$

The generalized inverse mean--free paths are given by the formula

$$\lambda_{mn} = \frac{4\pi}{iK} n_0 f_{mn}^{(0)} (1+\varepsilon_c) \qquad (22)$$

with n_0 being the constant nuclear density. The matrix Λ is constructed with λ_{mn} as its elements, meanwhile the matrix Δ being a diagonal matrix with the elements

$$\Delta = \mathrm{diag}(\Delta_1, \ldots \Delta_n) = \mathrm{diag}(k_1 - k, \ldots K_n - k) \qquad (23)$$

This completes the procedure of obtaining the matrix ($\Lambda - 2i\Delta$). It remains to calculate the matrix function G ($\Lambda - 2i\Delta$) = G (x). It was performed as follows:

The eigenvalues and the eigenvectors of the matrix X were first obtained, the diagonal matrix D was constructed with the eigenvalues along the diagonal and the matrix T was formed using the obtained eigenvectors. Writing

$$X = TDT^t \qquad (24)$$

with T^t being the transpose of T, the matrix function G (X) reduces to

$$G(X) = T\left[3D^{-3}\left[(1+D)e^{-D} - 1 + \frac{D^2}{2}\right]\right]T^t \qquad (25)$$

with the matrix between the outside brckets being a function of a diagonal matrix easy to calculate.

The calculation is confined to the isovector transverse photon component assuming the existence of an infinite sequence of I pc $= 1^{--}$ vector mesons. The contribution of the isoscalar and longitudinal components is neglected as in Ditsas & Shaw[6] and Ditsas[7], 10 vector - mesons are incorporated in the calculation. The contribution of the remaining vector mesons is considered to bea non - shadowing component.

Rising Cross - section: The fact that the cross–section is not asymptotically constant but rather increasing with energy (logarithmic increase), has been accounted for in this shadowing calculation with diagonal GVMD model. The adopted values of the cross–sections in the high--energy (40 – 100 GeV) kinematic region are

$$\sigma_{\rho N} = \frac{1}{2}\left(\sigma_{\pi^+ p} + \sigma_{\pi^- p}\right) = 22.4 + 13.5\,\nu^{-\frac{1}{2}} \quad mb$$

$$\sigma^{tot}_{\gamma N} = 110 + 58.5\,\nu^{-\frac{1}{2}} \quad \mu b \tag{26}$$

in agreement with the data from Cladwell *et. al.*[4].

This gives the transverse photon cross--section the value

$$\sigma_{\gamma N} = 86.8 + 58.5\,\nu^{-\frac{1}{2}} \quad (I_\gamma = 1) \tag{27}$$

The effect of increasing the cross--section on the shadowing calculation is discussed later.

Nuclear Density Distribution: In this calculation the nucleus is treated as a uniformly dense sphere with constant nuclear density given by

$$n_o = \frac{A}{\frac{4}{3}\pi R^3} = \frac{3}{4\pi r_o^3} \tag{28}$$

where $R = r_o \; A^{\frac{1}{3}}$

It has been a tradition to adopt r_o = 1.3 fm in the relation $R = r_o A^{\frac{1}{3}}$ for constant nuclear densities. The traditional choice arised from the fact that the scattering of nucleons with several MeV on nuclei may be approximately fitted, if nuclei are treated as uniform optical potentials with $R = r_o \cdot A^{\frac{1}{3}}$

In this calculation, r_o is fixed by asking the shadowing calculation for constant nuclear densities with off - diagonal GVMD model to agree with those for realistic nuclear densities with off - diagonal GVMD model of Ditsasa & and Shaw[6], and Ditsas[7]. The values found of r_o for the three studied nuclei are:

r_o (C) = 1.66 fm

r_o (Cu) = 1.46 fm

r_o (Pb) = 1.32 fm

The lack of more economic solutions (in terms of computer time) for the off--diagonal GVMD model with realistic densities motivates the above procedure.

RESULTS AND DISCUSSION

The main results are shown in Figures 1 to 7. The solid line is the prediction of off--diagonal generalized vector meson dominance (GVMD) model while the dashed line is that corresponding to diagonal GVMD model. In Figures 1 to 3 we show the energy dependence of shadowing for real photons incident on the well--studied nuclei, copper, carbon and lead, respectively. It is clear that shadowing does exist. The results of off--diagonal (GVMD) model lie above the data for very high energies. On the other hand, the diagonal GVMD model gives nice results for shadowing of high energy photons. The

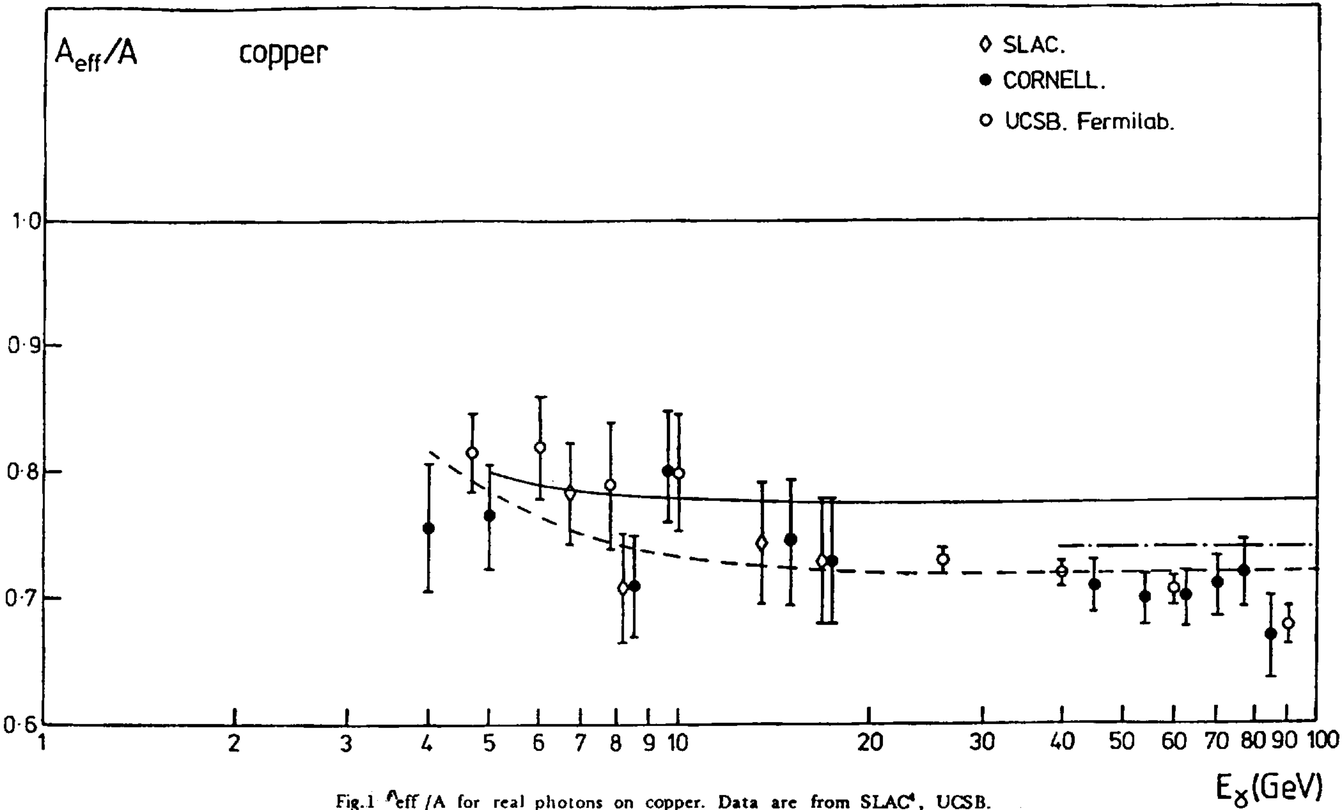

Fig. 1.

Fig.1 Aeff /A for real photons on copper. Data are from SLAC[4], UCSB. Fermilab(10) and CORNELL[1].

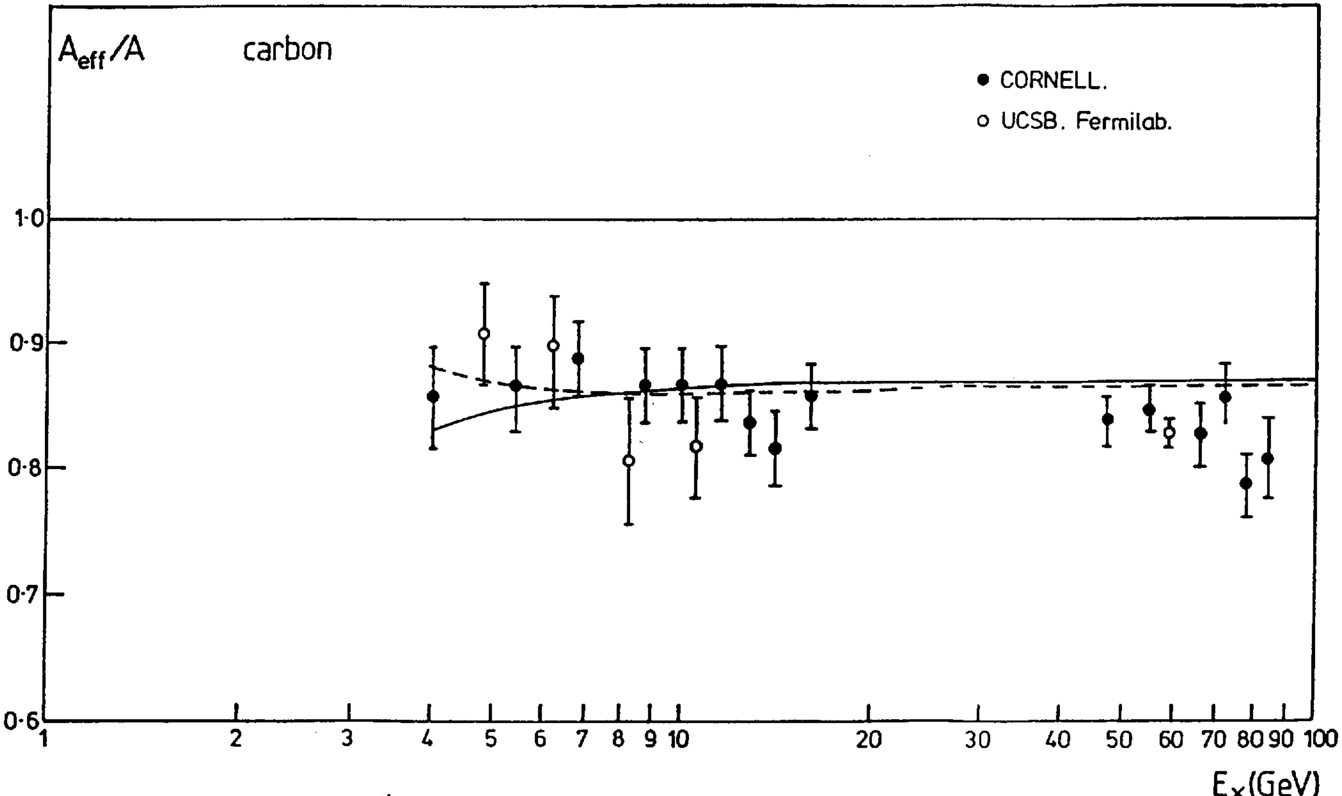

Fig. 2.

Fig. 2. A_{eff}/A for real photons on carbon. Data as in Fig. 1.

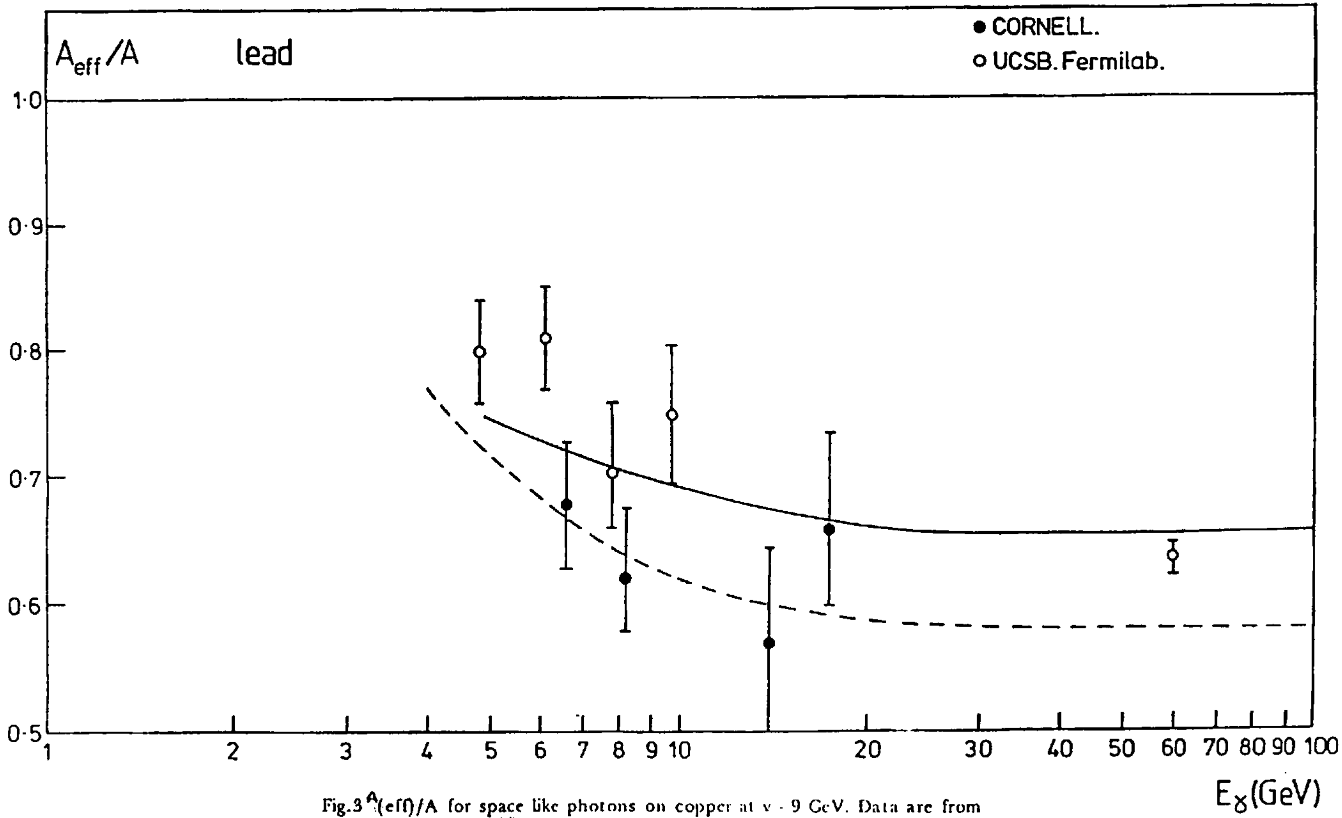

Fig. 3.

Fig.3 A(eff)/A for space like photons on copper at ν = 9 GeV. Data are from UCSB[3], SLAC[2,4] and CORNELL[1]

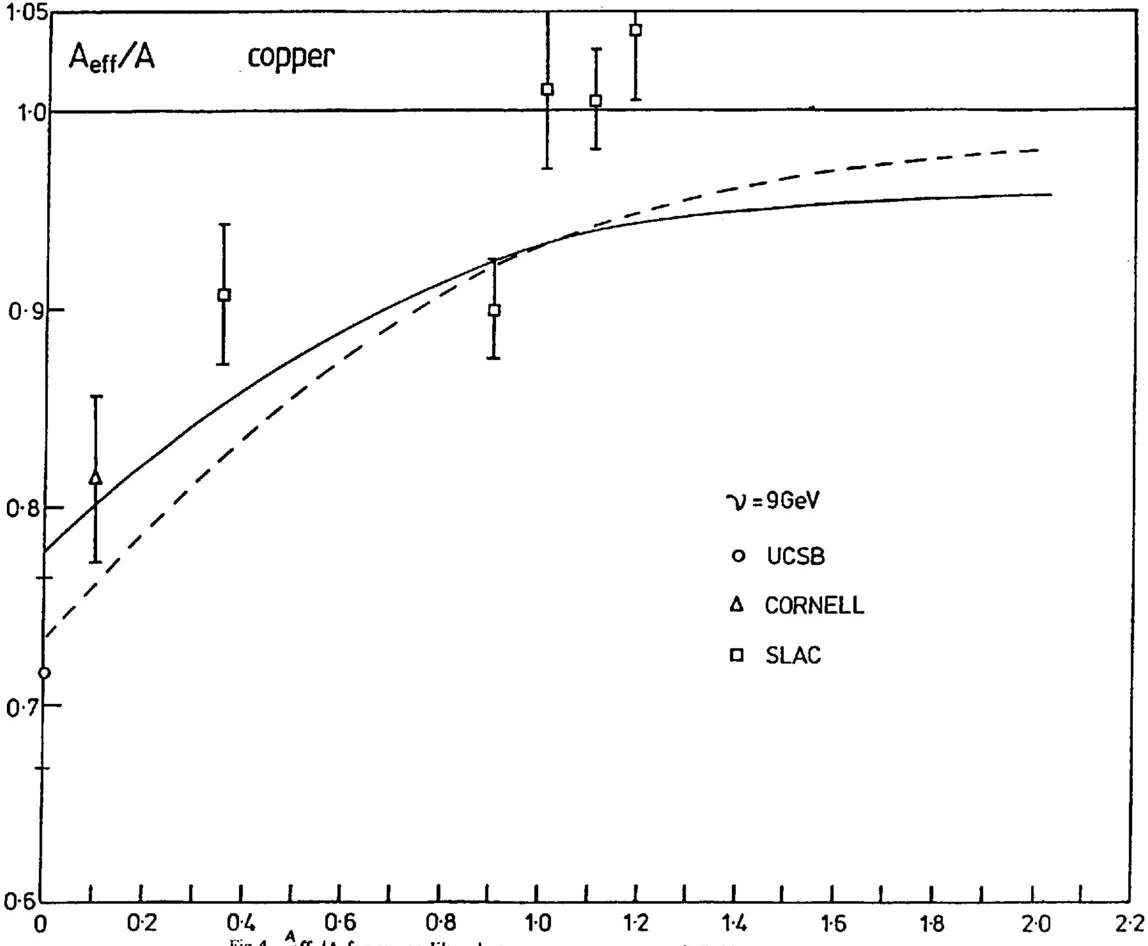

Fig. 4.

Fig.4. A_{eff}/A for space like photons on copper at $\nu = 9$ GeV. Data are from UCSB[3], SLAC[2,4] and CORNELL[9].

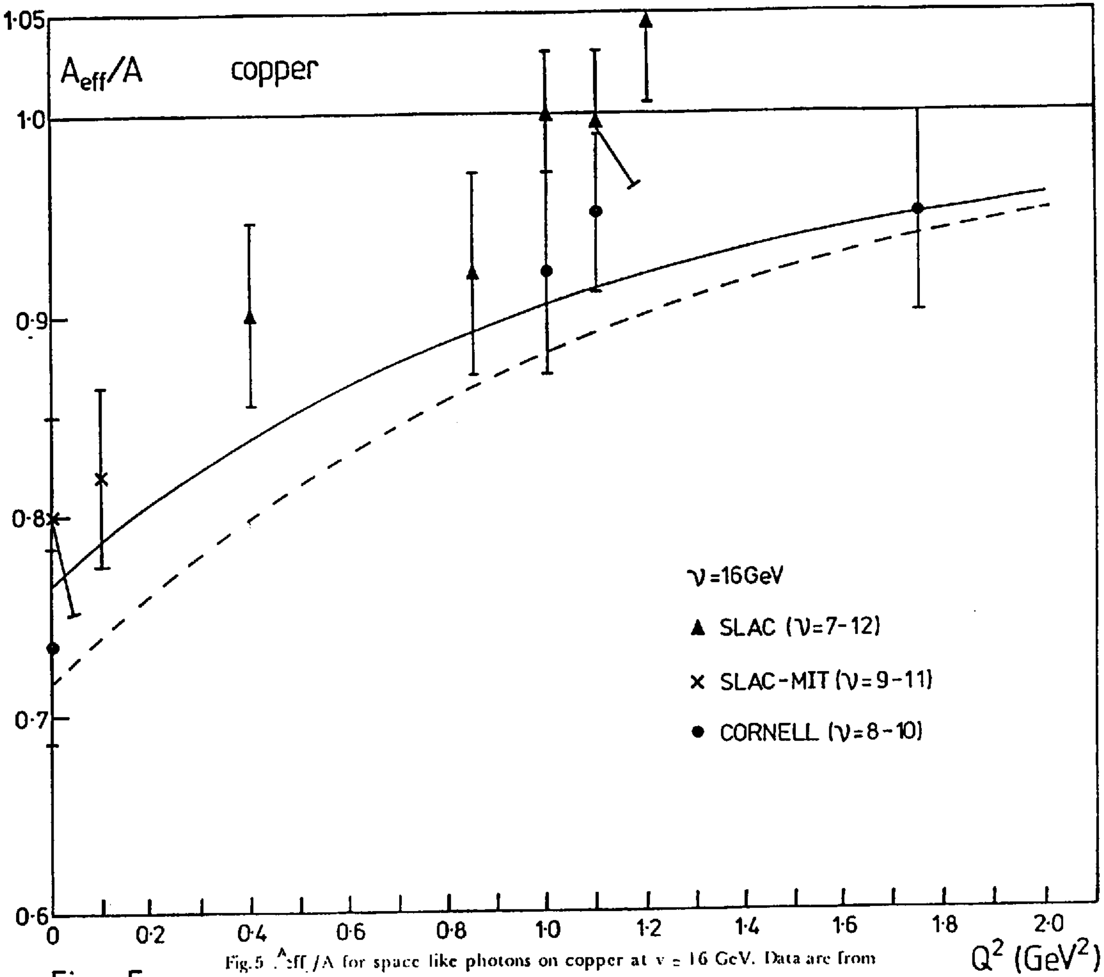

Fig. 5.

Fig.5 A_{eff}/A for space like photons on copper at ν = 16 GeV. Data are from SLAC[24], SLAC – MIT[14] and CORNELL[9].

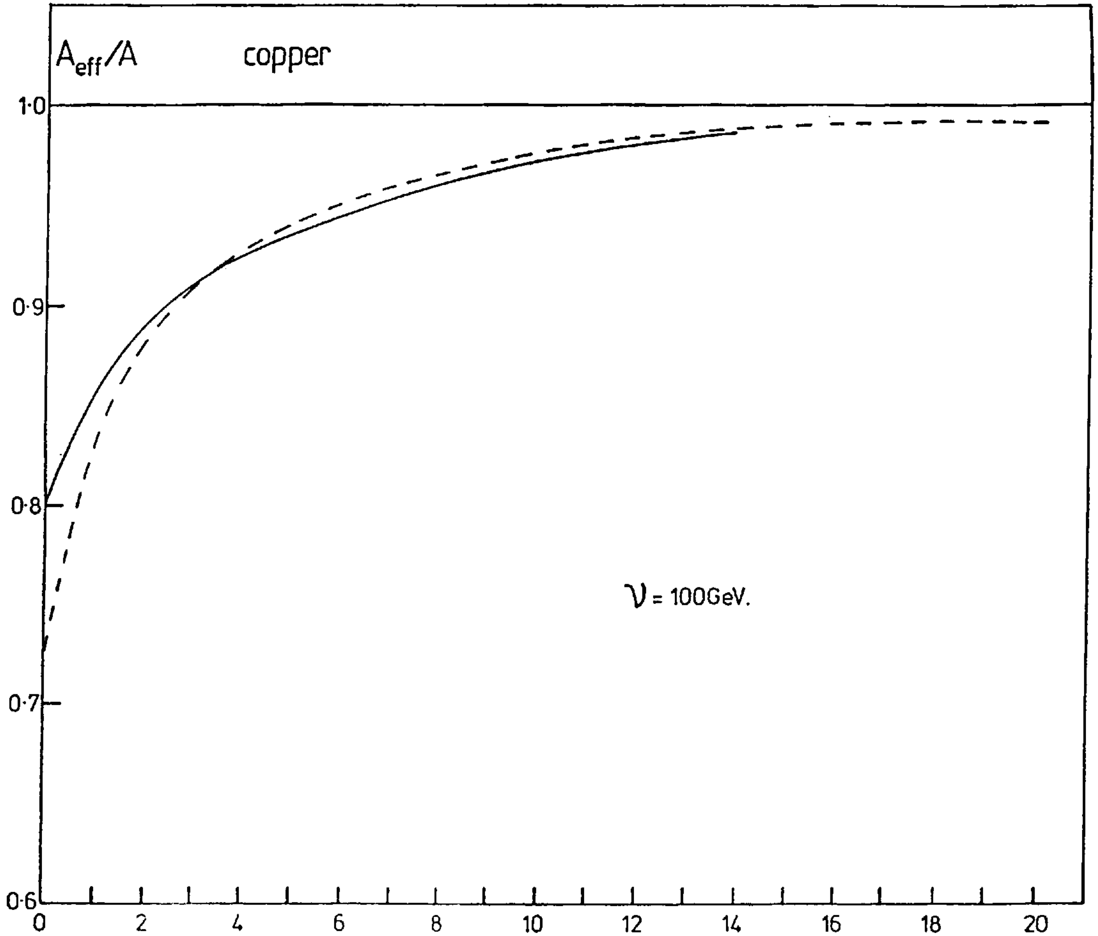

Fig. 6. Fig.6 A_{eff}/A for space-like photons on copper at $\nu = 100$ GeV. Q^2 (GeV^2)

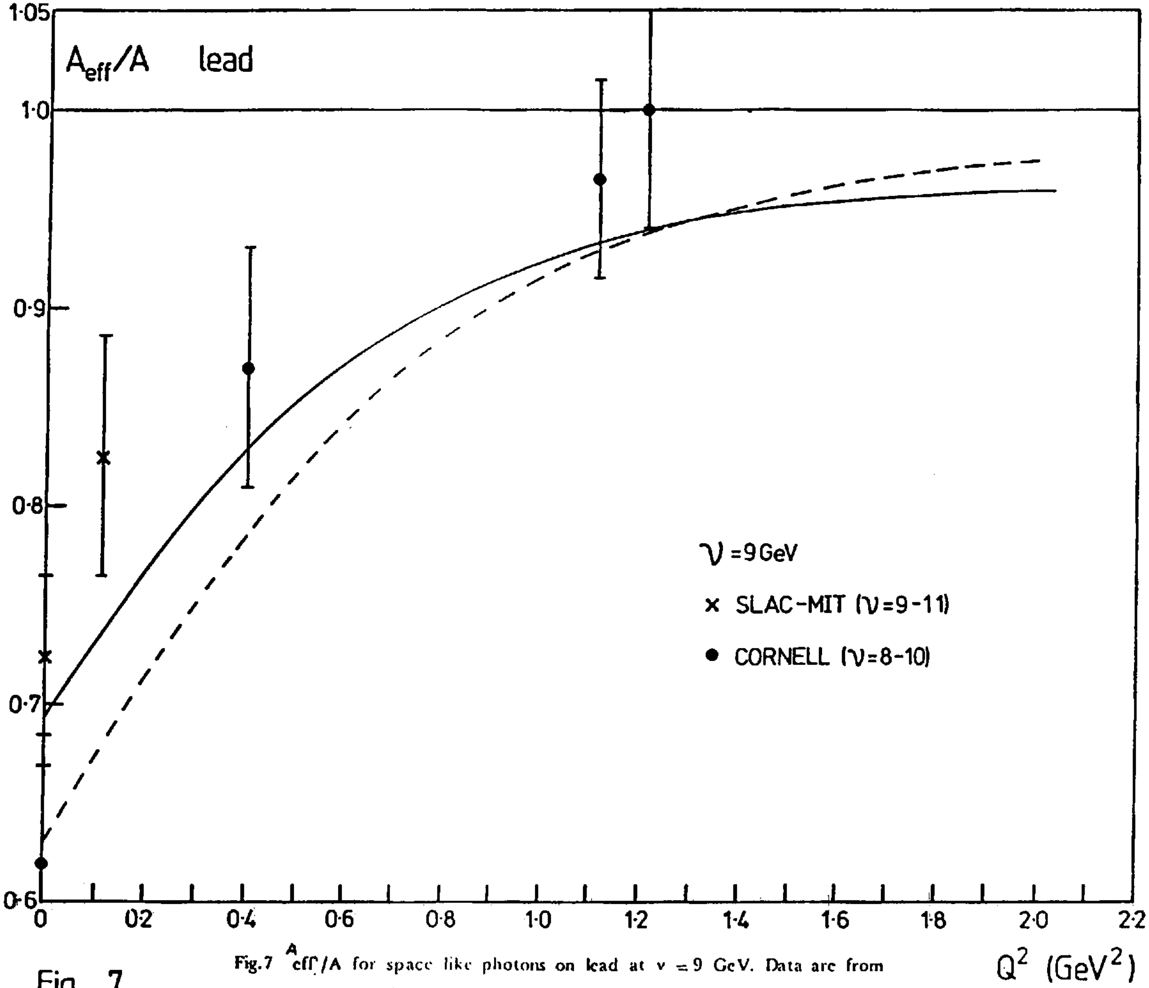

Fig. 7.

Fig.7 A_{eff}/A for space like photons on lead at $\nu = 9$ GeV. Data are from SLAC – MIT[4] and CORNELL[9].

prediction of both GVMD models is approximately the same for carbon at very high energies. We see that for Cu, C, and Pb, the off - diagonal GVMD gives an approximate 7%, 6% and 11% increase compared to diagonal GVMD, in A_{eff}/A for $\nu \geqslant$ 20 GeV. Also it is clear that shadowing does not increase appreciably with energy above 10 GeV. Bearing in mind the quantitative uncertainties ($\sim$ 5%) in the input parameters for shadowing calculation (e.g. $\sigma(nN)$, $\sigma(\gamma N)$, f(n) and L_c, etc) and the systematic uncertainties ($\sim$ 8 - 9%) in the data (mainly due to all sorts of radiative corrections), the results are compatible with experiment.

As Q^2 increases (at fixed ν) shadowing is expected to decrease as one can see from the coherence condition, namely

$$2\nu \geqslant (m_n^2 + Q^2) L_n$$

and because of the increasing relative importance of heavier vector mesons. In Figures 4 to 6 we show the Q^2 - dependence for Cu at ν = 9, 16, and 100 GeV respectively. Figure 7 gives the Q^2 - dependence for Pb at ν - 9 Gev. It is apparent (Figs 4--7) that off--diagonal GVMD model shows less shadowing at $Q^2 = 0$ than the diagonal GVMD but both models give the same prediction at high Q^2 ($\sim$ 14 GeV), as one can see from Figure 6.

The experimental data show the expected qualitative behaviour; shadowing decreases as Q^2 increases. However, the quantitative situation is not clear; there is still a discrepancy and the experimental data lie generally above the theoretical curves. Experimentally, a more rapid turn--off shadowing with Q^2 is observed but is not conclusively established because of the obscuring role played by radiative corrections and because of discrepancies among the various experiments. Existing data are not sufficiently precise to distinguish among VMD models. However better data over a wide kinematic region would be quite useful in testing the VMD models.

Finally, we turn to discuss the effect of increasing the cross section on shadowing within the diagonal (GVMD) model. Increasing the total vector--meson nucleon cross section would tend to decrease L_n i.e., $L_n \propto 1/\sigma(nN)$, and thus shadowing is expected to increse. In contrast, the calculation showed a decrease in shadowing. This is because of the increasing relative importance of increasing $\sigma(\gamma N)$. The prediction is shown in Figure 1 with dot--dashed line and shows an increase in A_{eff}/A (decrease of shadowing) of the order of 2.7% over the corresponding calculationwith the cross sections given by equations 8 and 10.

CONCLUSIONS

Nuclear shadowing of electromagnetic processes has been observed by several experiments and these experiments showed qualitative agreement with the predicted behaviour of shadowing (behaviour of shadowing with ν and Q^2). Existing data are not precise (high systematic uncertainties estimated to be of the order 8 – 9%) or enough to unambiguously select a particular theoretical model and inconsistencies among experiments make it impossible to draw any firm conclusions about the various models, but it seems that the high energy data points favours the diagonal (GVMD) model. Further and more precise data from experiments with real photons on nuclear target, even in the already studied kinematic region would provide valuable information.

The behaviour of shadowing with Q^2 is a critical test because it can distinguish between light--mass photon constituents from heavy ones. Experiments with high--energy virtual photons in the region $Q^2 \gg 1$ GeV^2 are capable of distinguishing between the general classes of vector dominance models. Furthermore, for large Q^2 one should take account of isoscalor and longitudinal photon components in the shadowing calculation. Finally, the results are compatible with data when systematic uncertainties are taken into account.

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