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Swing Up a Pendulum by Energy Control

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ABSTRACT

The inverted pendulum is a common-interesting control problem that involves many basic elements of control theory. This paper investigates the swinging up problem of a real pendulum from its lower position to the upper position and the balancing problem of the pendulum around the upper position. For swinging up the pendulum a fuzzy logic controller with two sets of rules, and two inputs is used while for stabilization a linear controller is used.

Keywords: *Inverted Pendulum, Swing up, Energy control, Fuzzy controller, Linear controller*

I. INTRODUCTION

Inverted pendulums have been used in control laboratories since the 1950s. Originally their use was mainly to illustrate ideas in linear control such as stabilization of unstable systems, Schaefer and Cannon (1967), Mori et el. (1976), Maletinsky et al. (1981), and Meir et al. (1990). Because of their nonlinear nature pendulums have maintained their usefulness and they are now used to illustrate many new emerging ideas

In this paper some properties of the simple strategies for swinging up the pendulum based on energy control will be investigated. Also stability control of the pendulum once in the upper position will be discussed. A real pendulum cart system is used. It includes the necessary equipment to constrain motion, apply force, measures states and implement control schemes. A block diagram of this system is shown in figure 1

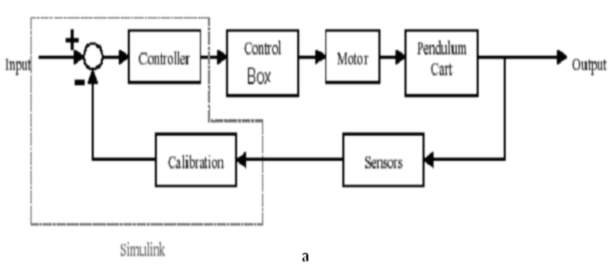

Figure 1 Block Diagram of the System

The ideas of energy control can be generalized in many different ways. Spong (1995) and Chung and Hauser (1995) have shown that it can be used also to control the position of the pivot.

II. MODELING THE PENDULUM-CART SYSTEM

A parametric model for the pendulum was derived. A parametric model is a transfer function or state variable description form, from which the poles and zeros of the plant can be obtained. The model consists of a number (pf) parameters such as coefficients of the polynomials, the element of the state description matrices, or the numbers that specify the poles and zeros.

Moment of inertia for the pendulum

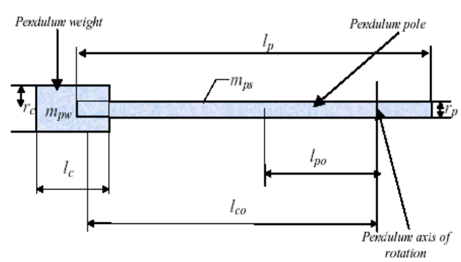

Figure 2 Pendulum setup

$$J = J_{aw} + J_{cw} + J_{cs} + J_{as} \quad (1)$$

Substituting the values in the above equation yields

$J = 0.01233$ [kg.m²]

Centre of mass for the pendulum and cart system

The value of center of mass for the Pendulum (x) can be calculated as follows:

$$x = \frac{m_{ps}\ \ell_{po} + m_{pw}\ \ell_{co}}{m_{ps} + m_{pw}} . \qquad \textbf{(2)}$$

Assuming that the mass of the cart is located at a point at the end of the pendulum stick, so the total distance between the center of mass of the pendulum and the point mass of cart is equals to (x) .The center of mass for the pendulum-cart set-up is located between the two points as shown in Figure3.

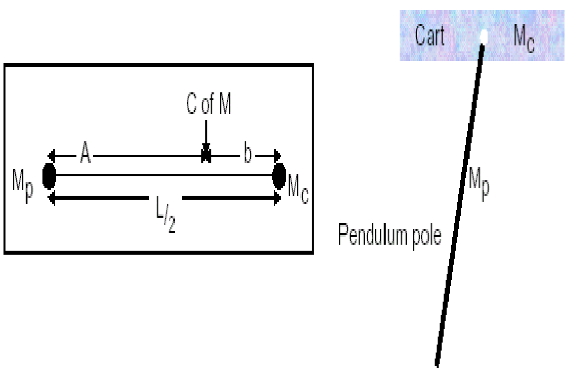

Figure 3: Pendulum pole

$$A = \frac{m_c\ x}{m_p + m_c} \qquad (3)$$

The center of mass for the pendulum-cart system is given by:

$L = x - A$ (*the* center of mass for the pendulum-cart system)

Substituting the values of constants yields a center of mass:

$L = 0.0295$ m.

Friction modeling

The experimental relationship between cart friction and cart velocity is shown in figure 4 Ableson , C . F . (1996)

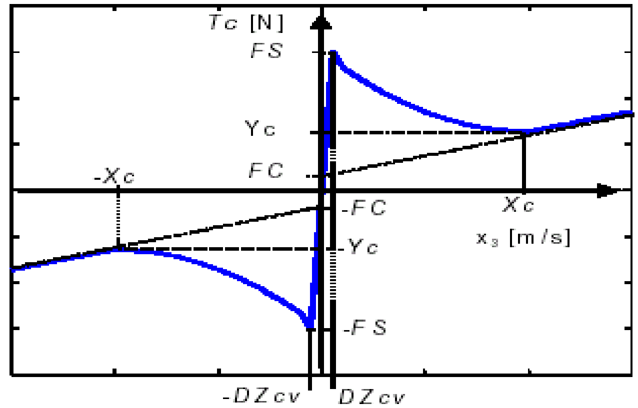

Figure 4 Experimental friction plots

Where

FS - static friction [N],
FC -dynamic or Coulomb friction [kg/s],
Xc - cart velocity - beginning of the linear dependence zone [m/s]

Yc -friction value for Xc point [N]
DZcv - dead zone of cart velocity [m/s].

In order to obtain some analytical results for the total friction force acting on the cart, the friction function is divided into 2 zones:

x_3< -DVcv, x_3 > DVcv (x_3 : cart velocity) Zone 1

-DVcv< x_3 <DVcv Zone 2

zone 1:

The friction in this zone can be approximated by an exponential function Ableson , C . F . (1996)

$$TC = \operatorname{sgn}(x_3)\ FC + \operatorname{sgn}(x_3)\left[FS - FC\right] \times \exp\left(\frac{-abs(x_3)}{x_c}\right)^n + x_3\ \frac{Y_c - FC}{X_c} \qquad (4)$$

zone 2:

The friction linearly increases in the range DVcv < x_3 < DVcv .From Figure 4 the value of the Friction at x_3 = ± DVcv is ±FS .Using a linear equation and noting that this linear

part passes through the origin, yields:

$$TC = \frac{FS\ x_3}{DZ_{cv}}. \qquad (5)$$

Figure 5 shows a plot of the approximated friction compared with the friction curve plot.

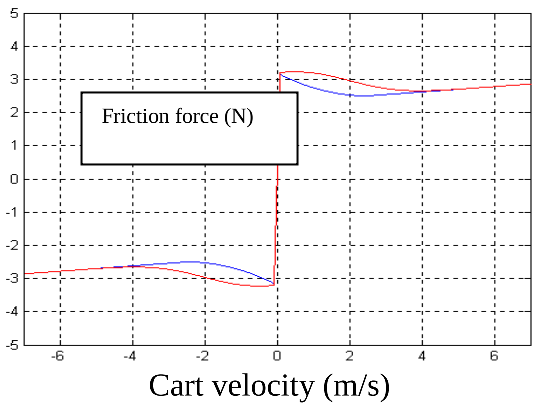

Figure 5: Approximated friction plot (in red) compared to friction curve

Modeling the Pendulum-Cart Set-Up

Linear Controller Design

The designed controller has two functions:

1) To gradually swing the pendulum to the inverted position
2) To balance the pendulum at the unstable equilibrium point. The swinging is done by a fuzzy controller or the position alternating controller. The stabilizing controller is designed by using the state feedback controller.

Defining a state vector as $X = [x_1\ x_2\ x_3\ x_4]^T$, the state space model will have the following form:

$$X' = Ax + Bu$$
$$Y = Cx + Du$$

$$\Rightarrow \quad X' = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{L\,\mu\, g}{J} & \frac{-a\, f_v}{J} & \frac{-L\, f_p}{J} \\ 0 & \frac{\mu\, g}{J} & \frac{-L\, f_v}{J} & \frac{-f_p}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{a}{J} \\ \frac{L}{J} \end{bmatrix} F$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} F$$

Substituting the values of the constants in the above matrices yields:

$$\Rightarrow \quad A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.8586 & -0.0716 & -2.57\times10^{-4} \\ 0 & 29.1 & -0.19 & -8.714\times10^{-3} \end{bmatrix}$$

$$,\quad B = \begin{bmatrix} 0 \\ 0 \\ 0.87659 \\ 2.3925 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

LQR – Controller

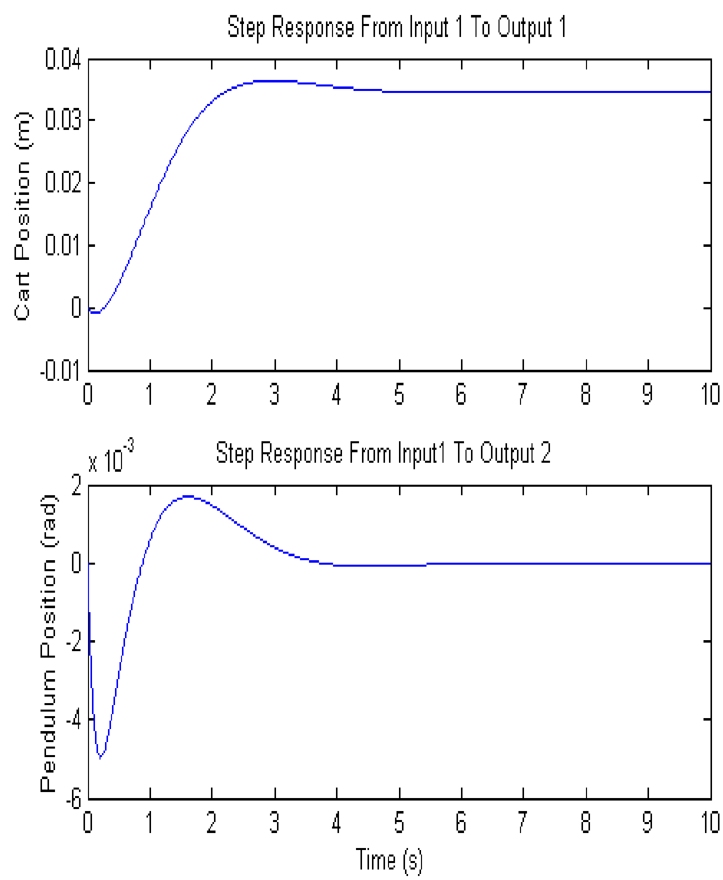

Figure 6: pendulum & cart position responses for step input and K trial # 1

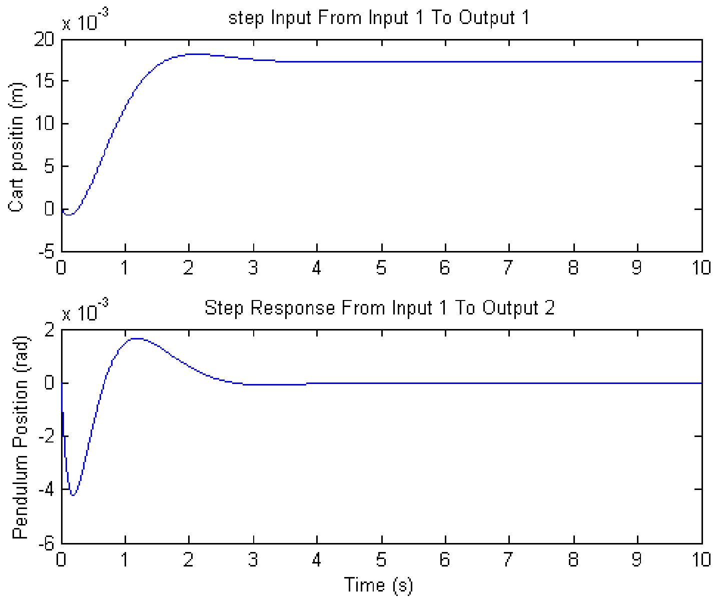

Figure 7: pendulum & cart position responses for step input and K of trial # 2

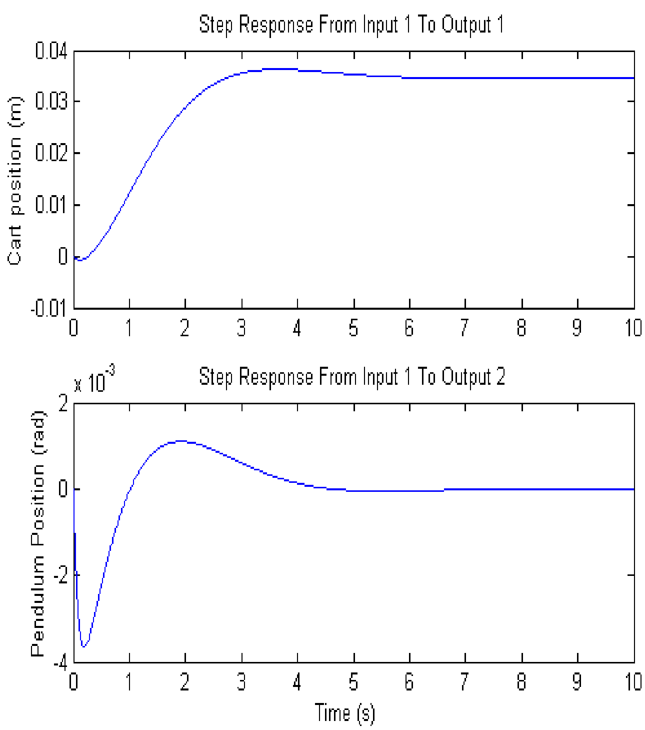

Figure 8: pendulum & cart position responses for step input and K of trial # 3

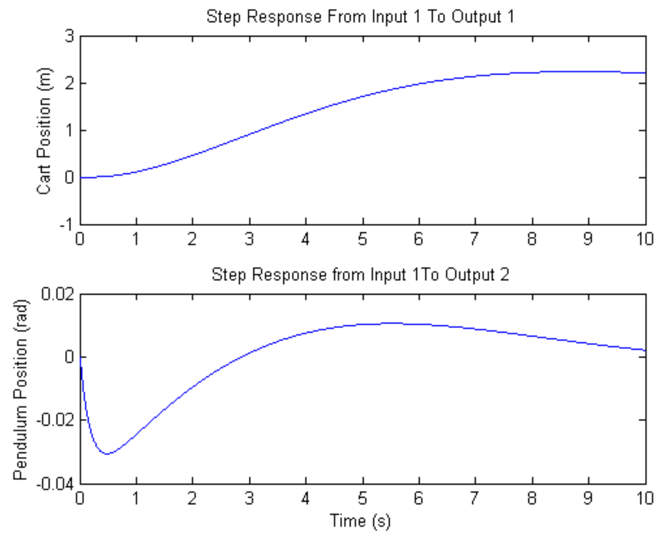

Figure 9: pendulum & cart position responses for step input and K of trial # 4

Performance of LQR – Controller:

The Evaluation Criteria are:

* Closed loop poles:

From the four trials, **Q** and **R** can be found. It is clear that by increasing **Q** the stability increases, this means that the closed – loop poles moves to the left away from the imaginary axis. But by increasing **R** the closed – loop poles move to the right, closer to the imaginary axis, which means that the controller losses some stability. By increasing the element Q (1, 1), the settling time decreases, but by increasing the element Q (2, 2) the settling time increases. By increasing **R** the settling time increases. After closely inspecting all 4 trials the results gained from trial # 1 Figure 1will be used as state – feedback gains in the linear controller, since it has the best response.

Nonlinear Swinging up controllers

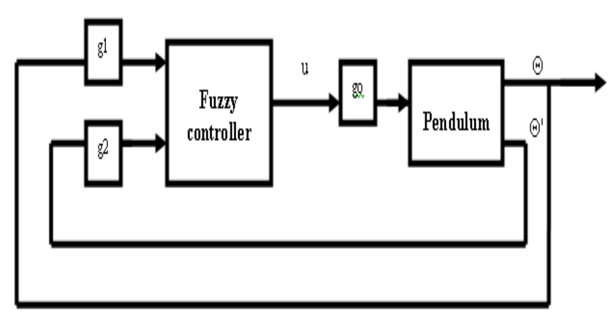

Figure 10: Block diagram for two inputs Fuzzy controller

The gains **g0, g1, g2** appearing in the block diagram are scaling gains that are used to change the width of the

To balance the pendulum on the up position two controllers are needed: First swing-up controller brings the pendulum near the linear zoon, and a linear controller to balance the pendulum at the upward position. Different approaches are used for swinging up the pendulum. One approach of swing up controller uses energy theory Astrom , K . J et al. (1995), Iwashiro , M et al. (1996), Astrom , K . J .(1999) , another uses a sliding controller as a swing up controller Furuta , K. and M. Yamakita (1991), Furuta , K. and M. Yamakita et al (1992). In this paper fuzzy control will be used to swing up the pendulum.

Swinging up the inverted pendulum using two inputs fuzzy controller:

A block diagram of this type of controller is shown in Figure 10; and the schematic diagram is shown in Figure 11. The inputs to the swing- up controller are (**Θ**, **Θ'**), and the output is **u**.

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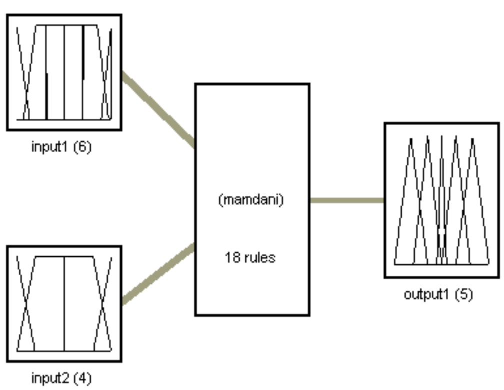


Figure 11: Schematic diagram of the Fuzzy controller (two inputs, one output, 18 rules)

Input variable **Θ** (measures with relative to downward position) has six memberships function symmetrical by center as shown in Figure 10 , (we used a rectangle shape for the **psmall** and **nsmall** since we don't' need to know the accurate value of angle **Θ** in this region

Input variable **Θ'** has four membership functions, that are symmetrical about the center as shown in Figure 11.

The output variable u shown in Figure 14 has five triangular membership functions that are squeezed at the center to get a refine overshoot.

memberships function directly without the need to change each membership function discourse.

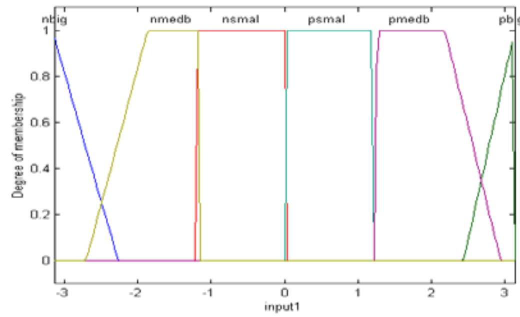


Figure 12: Membership function for input Θ

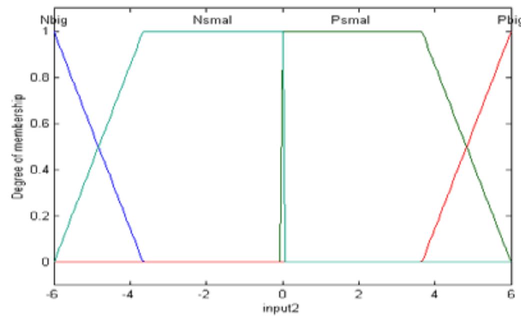


Figure 13: Membership function for input Θ'

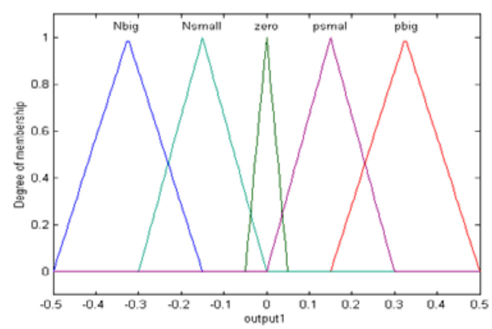


Figure 14: Membership function for output u

The rules for the swing-up fuzzy controller can be described as follows

1- For small angle $|\Theta| < 90°$.

If angle **Θ** is positive (negative) and if angle velocity **Θ'** is positive (negative) then the

Output **u** is positive (negative). This strategy will increase the energy of the pendulum.

2- For medium angles the output **u** is zero.

3- For big angles close to the upper position:

If the angle velocity is small, and it has the same sign like the angle **Θ** then the energy must be increased like in case 1.

If the angle velocity is big, and it has the same sign like angle **Θ**, the output **u** is zero [12]

These rules are implemented as shown in table 1.0

Table 1.0: Rules for Two Inputs Fuzzy System

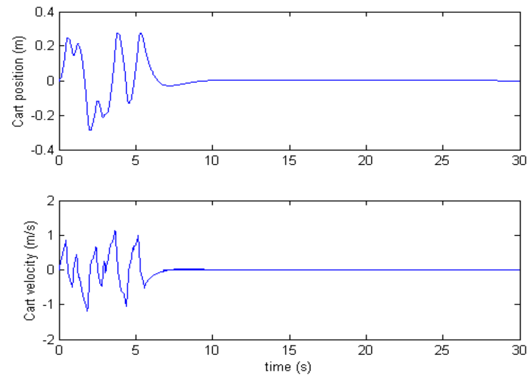
Θ' / Θ	Nbig	Nsmall	Psmall	Pbig
Nbig	1	-1	-2	0
Nmed	0	0	0	0
Nsmall	2	2	-2	0
Psmall	0	2	-2	-2
Pmed	0	0	0	0
Pbig	0	2	1	-1

Where:

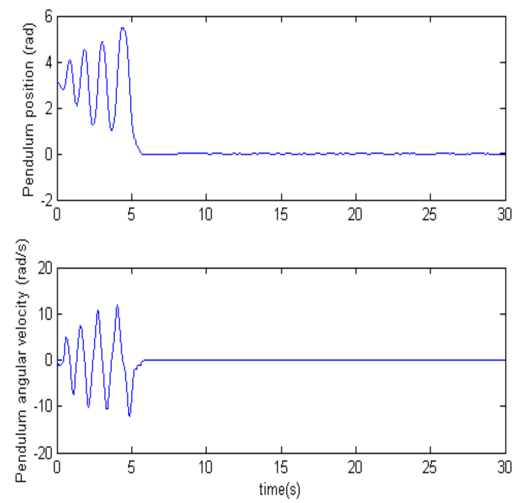
(Positive 1) denote positive small.
(Positive 2) denote positive medium relative to **Θ** and (positive large for **Θ'** and **u.**
(Positive 3) denote positive large relative to **Θ**
(Negative 1) denote negative small.
(Negative 2) denote negative medium relative to **Θ** and (positive large for **Θ'** and **u**.
(Negative 3) denote negative large relative to **Θ**

Simulation and Experimental Results for the two input fuzzy controller:

Figure 15 shows the simulation results that are obtained using the two inputs Fuzzy controller. It can be seen from Figure 15-a that the cart position and velocity approach zero after the pendulum enter the linear zoon .Figure 15-b shows that the pendulum position at the beginning is **π**, but after the singing –up controller is enabled the position began to increase and decrease in a way that added energy to the pendulum until it reach the upward unstable position where the linear controller stabilized it a round the zero.



(a)



(b)

Figure 15: Simulation results of the two inputs Fuzzy

CONCLUSION

Energy control is a very convenient way to swing up a pendulum. The results clearly show that the 2 input fuzzy controls and the linear controller worked as expected. These controllers were able to swing and maintain the pendulum at an upright position.

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