Relativistic study of the energy-dependent Coulomb potential including Coulomb-like tensor interaction

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Abstract: The exact Dirac equation for the energy-dependent Coulomb (EDC) potential including a Coulomb-like tensor (CLT) potential has been studied in the presence of spin and pseudospin symmetries with arbitrary spin–orbit quantum number, \( \kappa \). The energy eigenvalues and corresponding eigenfunctions are obtained in the framework of the asymptotic iteration method. Some numerical results are obtained in the presence and absence of EDC and CLT potentials.

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Résumé : Nous étudions l’équation de Dirac exacte pour le potentiel de Coulomb dépendant de l’énergie (EDC), incluant un potentiel tensoriel de type Coulomb (CLT), en présence des symétries de spin et de pseudo-spin avec nombre quantique de spin–orbite arbitraire. Les valeurs et fonctions propres sont obtenues via la méthode d’itération \( \kappa \) asymptotique. Certains résultats numériques sont présentés avec et sans potentiels EDC et CLT.

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1. Introduction

In the framework of the Dirac equation, the pseudospin (p-spin) symmetry is usually used to feature deformed nuclei, superdeformation, and to establish an effective shell-model [1–3], whereas the spin symmetry is relevant for mesons [4]. Furthermore, the spin symmetry occurs when the difference of scalar potential \( S(r) \) and vector potential \( V(r) \) is constant (i.e., \( \Delta(r) = S(r) - V(r) = C_s \) whereas the p-spin symmetry occurs when the sum of scalar and vector potentials is constant (i.e., \( \Sigma(r) = S(r) + V(r) = C_p \)) [5, 6]. About 40 years ago, the p-spin concept was considered for the first time in the nonrelativistic framework [7, 8]. The p-spin symmetry refers to a quasi degeneracy of single nucleon doublets with nonrelativistic quantum number \( (n, l, j) = (l + 1/2) \) and \( (n - 1, l + 2, j = l + 3/2) \), where \( n, l, \) and \( j \) are single nucleon radial, orbital, and total angular momentum quantum numbers, respectively. The total angular momentum is \( j = \tilde{l} + \tilde{s} \), where \( \tilde{l} = l + 1 \) is a pseudoangular momentum and \( \tilde{s} \) is p-spin angular momentum [9–19]. Also, tensor potentials were introduced into the Dirac equation with the replacement \( p \rightarrow p - i\alpha \cdot \mathbf{R} U(r) \) and a spin–orbit coupling is added to the Dirac Hamiltonian [20–30].

Wave equations with energy-dependent potentials occur in relativistic quantum mechanics, firstly with the Pauli–Schrödinger equation [31] and recently in the hamiltonian formulation of the relativistic many-body problem [32–34]. Also, energy-dependent potentials have been used as a source of the nonlinear hamiltonian evolution equation [35–38] and are currently applied to soliton propagation [39–41]. Recently, extensive studies on energy-dependent potentials have appeared in some recent works [42–46].

The aim of the present work is to study the Dirac equation for the attractive scalar and repulsive vector energy-dependent Coulomb (EDC) potential including the Coulomb-like tensor (CLT) potential under the p-spin and spin symmetric limit. We solve the relativistic equation to obtain its bound state solutions including the energy eigenvalues and the corresponding wave functions by means of the asymptotic iteration method (AIM) [47–51].

The structure of this paper is as follows. In Sect. 2, we briefly present the AIM. In Sect. 3, the Dirac equation with EDC scalar and vector potentials including the CLT potential is briefly introduced. We solve the Dirac equation under p-spin and spin symmetric limits and give some numerical results too. Finally, our concluding remarks are given in Sect. 4.

2. AIM

The AIM has been proposed to solve second-order differential equations having the form

\[
\frac{d^2 y(x)}{dx^2} = \lambda_0(x) \frac{dy(x)}{dx} + S_0(x) y(x)
\]  

(1)

where \( \lambda_0(x) \neq 0 \) and the variables \( \lambda_0(x) \) and \( S_0(x) \) are sufficiently differentiable functions [47–51]. Differential equation (1) has a general solution given as follows: