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Reducing ICI effect in OFDM system using low-complexity Kalman filter based on comb-type pilots arrangement

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SUMMARY

In OFDM systems, time-varying channels destroy orthogonality between subcarriers causing Inter-Carrier Interference. To reduce this effect, a Kalman filter, as a benchmark, is used for channel estimation, based on comb-type pilot arrangements of the OFDM system. An advantage of comb-type pilot arrangements in channel estimation is the ability to track the variation of the channel caused by Doppler frequency. Kalman method has been proposed to estimate the channel frequency response (CFR) at the pilot locations, then CFR, at data subchannels, is obtained by means of interpolation between estimates at pilot locations. The low-complexity Kalman method is introduced to reduce the complexity of the system while achieving the same BER/SNR. Different types of interpolations have been also compared such as Low-pass, Spline-cubic and Linear interpolation methods. The BER/SNR performance of BPSK modulation schemes are considered for time varying Rayleigh fading channels. Our results has shown that the low-complexity Kalman estimation, used with the pilot arrangement and a suitable interpolation method, gives almost the same performance as that of the Kalman method specially for low SNR values and hence the effect of Doppler shift effect is controlled. Copyright © 2010 John Wiley & Sons, Ltd.

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KEY WORDS: channel estimation; OFDM; time-varying channels; Kalman filtering; ICI; Rayleigh fading

1. INTRODUCTION

Orthogonal frequency division multiplex (OFDM) is an effective technique for combating frequency-selective fading channels in wireless communication systems; OFDM divides the overall frequency band into a number of sub-bands and transmits a low-rate data stream in each sub-band. In this way, a wideband frequency-selective channel is converted to a number of parallel narrow-band flat-fading subchannels, which are free of Inter-Symbol Interference (ISI). In addition, OFDM allows overlap of the subchannels but keeps the orthogonality of the subcarriers. Therefore, high spectral efficiency is achieved. For coherent detection of the information symbols, reliable estimation of the gain of each subchannel in the OFDM system is crucial. This problem is further complicated by the time-varying nature of the channel fading and the correlation between the subchannels due to Doppler frequencies [1].

OFDM-based systems are generally used in time-varying frequency-selective fading channels. In this case, the channel transfer function changes across the subcarriers and OFDM symbols. Therefore, pilot subcarriers are distributed uniformly in time and frequency axes [2]. The channel coefficients that belong to the pilot subcarriers are estimated. In comb-type-based channel estimation, the pilots are inserted into every OFDM symbol in order to track the time-varying nature

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of the channel. This will reduce the complexity of the estimation methods since the time domain interpolator is not needed. Different estimation techniques may be used (like least square (LS), minimum mean-square error (MMSE), least mean-square (LMS) or the Kalman estimation); then, these estimates are interpolated using a suitable method (like Low-pass interpolation, Spline-cubic interpolation or Liner interpolation) to estimate the CFR at the data subchannels [2].

This paper is concerned with reducing the Inter-Carrier Interference (ICI) in a comb-type OFDM system; this work has been previously discussed using several approaches without using the complex Kalman filter technique such as using a proposed pulse shape [3], or a partial transmit sequence and selected mapping [4], or improved sinc power [5] and 'better than' raised cosine pulse [6]. However, in this paper, we have used the Kalman filter method as a reference point, to estimate the desired signals; then a low-complexity Kalman filter approach is utilized as an estimation method followed by a suitable interpolation technique under certain environment to reduce the ICI.

The remainder of the paper is organized as follows. Section 2 describes the general OFDM system model. In Section 3, the comb-type pilot arrangements of OFDM system is discussed. The adaptive Kalaman filter is discussed in Section 4 as a reference. The low-complexity Kalaman estimator is discussed in Section 5. The simulation environment and the results are presented in Section 6 and finally, Section 7 concludes this work.

2. OFDM SYSTEM MODEL

In standard OFDM system, the information symbols are grouped into blocks. Then inverse discrete Fourier transform (IDFT) is performed on each block and a proper cyclic prefix (CP) extension is added before they are fed into the modulator and transmitted. At the receiver side, discrete Fourier transform (DFT) is performed on each received OFDM symbol after the CP is removed.

The OFDM system based on pilot channel estimation is given in Figure 1 [7]. The binary information is first grouped and mapped according to the modulation in signal mapper. After inserting pilots either to all sub-carriers with a specific period or uniformly between the information data sequence, IDFT is used to transform the data sequence of length N $\{X(k)\}$ into time domain

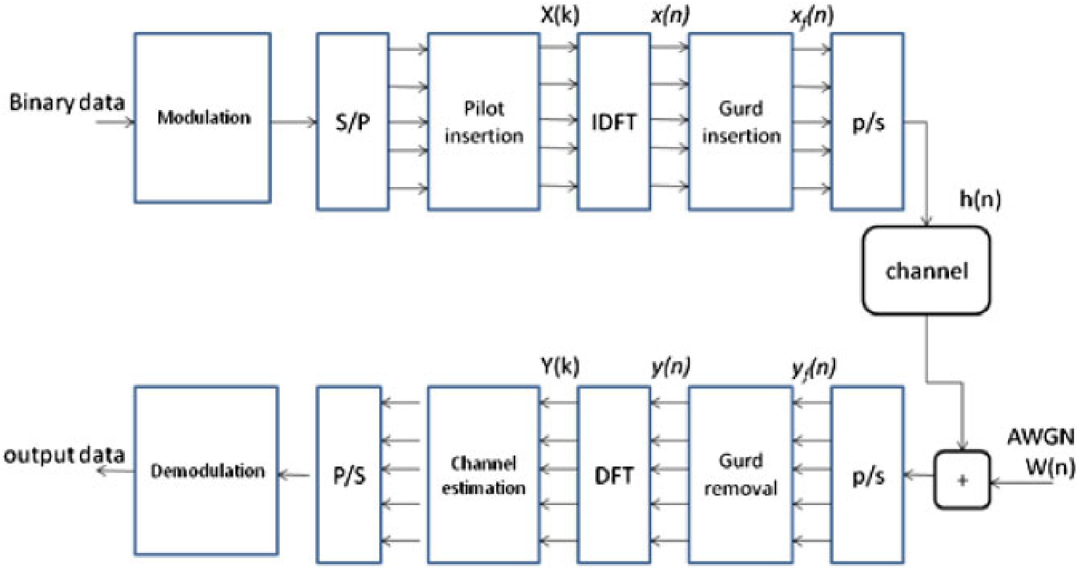

Figure 1. OFDM system model.

signal $\{x(n)\}$ as

$$x(n)=\sum_{k=0}^{N-1} X(k)\mathrm{e}^{\mathrm{j}(2\pi kn/N)}, \quad n=0,1,2,3,\ldots,N-1 \tag{1}$$

where N is the DFT length.

Following IDFT block, guard time, which is chosen to be larger than the expected delay spread, is inserted to prevent ISI. This guard time includes the cyclically extended part of OFDM symbol in order to eliminate ICI [8]. The resultant OFDM symbol is given by

$$x_f(n)=\begin{cases} x(N+n), & n=-N_g,-N_g+1,\ldots\ldots,-1 \\ x(n), & n=0,1,2,3,\ldots.,N-1 \end{cases} \tag{2}$$

where N_g is the length of the guard interval.

The transmitted signal $x_f(n)$ will pass through the time-varying frequency-selective fading channel with additive noise. The received signal is given by

$$y_f(n)=x_f(n)\otimes h(n)+w(n) \tag{3}$$

where $w(n)$ is the additive White Gaussian noise (AWGN) and $h(n)$ is the time-varying channel impulse response given as

$$h(n)=\sum_{i=0}^{L-1} h_i\,\mathrm{e}^{\mathrm{j}(2\pi/N)f_{\mathrm{d}i}T_n}\delta(\tau-\tau_i), \quad 0\leqslant n\leqslant N-1 \tag{4}$$

where L is the total number of propagation paths, h_i is the complex impulse response of the ith path, $f_{\mathrm{d}i}$ is the ith path Doppler frequency shift, τ is delay spread index, T_n is the OFDM duration time and τ_i is the ith path delay normalized by the sampling period T_{s}.

At the receiver side, the guard period is removed, yielding $y(n)$; then the signal is fed into a DFT function obtaining $Y(k)$ as [7]

$$Y(k)=\mathrm{DFT}(y(n))=X(k)H(k)+W(k)+I(k) \tag{5}$$

where $H(k)$ is the channel frequency response (CFR) given as

$$H(k)=\sum_{i=0}^{L-1} h(n)\mathrm{e}^{-\mathrm{j}(2\pi kn/N)}, \quad k=0,1,2,3,\ldots,N-1 \tag{6}$$

And $W(k)$ is an AWGN given as;

$$w(k)=\frac{1}{N}\sum_{n=0}^{N-1} w(n)\mathrm{e}^{-\mathrm{j}(2\pi nk/N)}, \quad k=0,1,2,3,\ldots,N-1 \tag{7}$$

The final term of Equation (5), $I(k)$, represents the ICI value caused by time-variant nature of the channel when the Doppler frequency is high, which is assumed to be a Gaussian random variable according to central limit theory [8]. Both $I(k)$ and $W(k)$ are having bad effects on the useful signal when the AWGN approaches zero (high SNR) and hence the total noise becomes dominantly ICI given as [9]

$$I(k)=\frac{1}{N}\sum_{m=0,m\neq k}^{N-1} x(m)\sum_{n=0}^{N-1} H(m)\mathrm{e}^{\mathrm{j}(2\pi n(m-k)/N)} \tag{8}$$

Following DFT operation, the pilots' signals are extracted and the estimated channel, $H_e(k)$, for the data subchannels, is estimated. Then the transmitted data, x_e, is estimated as

$$x_e=\frac{Y(k)}{H_e(k)} \tag{9}$$

3. COMB-TYPE PILOT DESCRIPTION

For comb-type pilot subcarrier arrangement, the K_p pilot signals $X_p(m)$, $m=0, 1, 2, \ldots, K_p$, are uniformly inserted into $X(k)$. That is, the total N subcarriers are divided into K_p groups, each with $L=N/K_p$ adjacent subcarriers. In each group, the first subcarrier is used to transmit pilot signal. The OFDM signal modulated on the kth subcarrier can be given by

$$x(k)=x(mL+l) \tag{10}$$

where

$$x(k)=\begin{cases} x_p(m)=0 & \text{where } l=0 \\ \text{inf. data} & \text{where } l=1,2,\ldots L-1 \end{cases} \tag{11}$$

$X_p(m)$ is the mth pilot carrier value and the received pilot signal vector $Y_p=[Y_p(0), Y_p(1), \ldots, Y_p(K_p-1)]^{\mathrm{T}}$ can be expressed as

$$Y_p=X_pH_p+W_p \tag{12}$$

where

$$X_p=\begin{bmatrix} X_p(0) & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & X_p(N_p-1) \end{bmatrix} \tag{13}$$

H_p is the frequency response of the channel at pilot subcarriers and W_p is a vector of the Gaussian noise, H_p and W_p can be expressed as

$$H_p=[H_p(0), H_p(1), \ldots, H_p(K_p-1)]^{\mathrm{T}} \tag{14}$$

$$W_p=[W_p(0), W_p(1), \ldots, W_p(K_p-1)]^{\mathrm{T}} \tag{15}$$

4. KALMAN ESTIMATOR

An adaptive estimator is the one that updates its parameters over time as a consequence of changing channel statistics. Kalman estimator is one of the adaptive estimation algorithms used to estimate the fading process in OFDM system. It is an efficient recursive algorithm that estimates the state of a dynamic system from a series of noisy measurements. It has been applied in communication systems since 1970s [10]. Kalman filter estimates the fading process by minimizing the estimation error variance $E[|H_p-\dot{H}_p|^2]$, where $\dot{H}_p$ is obtained using the Kalman filter. It could also be applied to track the states of time-varying channels where a pth order AR model is presented as [11]

$$H_p(l+1)=\sum_{i=0}^{p-1} F[i]H_p(l)+v(l) \tag{16}$$

$F[i]$ and the variance of the noise can be obtained by solving the YW equation using the ACF of the fading process. The correlation of the time-variant channel is modeled as

$$E[h(l,i)h^*(l+1,j)]=\begin{cases} P_i J_0(2\pi F_{\mathrm{d}}T_{\mathrm{s}}), & i=j \\ 0, & i\neq j \end{cases} \tag{17}$$

where $P_i=[Eh(l,i)]^2$, F_{d} is the maximum Doppler shift, T_{s} is OFDM symbol time, $J_0(\cdot)$ is the Bessel function of the first-kind and zero order, and i, j are the frame and subcarrier index, respectively. Using $H_p(l)=Wh(l)$,

$$E[H(l)^H H(l+1)]=J_0(2\pi F_{\mathrm{d}}T_{\mathrm{s}})WPW^{\mathrm{T}} \tag{18}$$

where P is a *NXN* diagonal matrix with diagonal elements P_i and W is a *PXN* partial DFT matrix obtained from a DFT matrix by deleting the rows that does not correspond to pilot symbols. Here, P denotes the number of pilot symbols in one OFDM symbol and N is the total number of symbols.

The state-space model for the Kalman filter state variable is defined in the form of channel states as

$$H_p(l+1)=\mathbf{F}H_p(l)+v_p(l) \tag{19}$$

where $H_p(l)$ is the frequency response of the channel and $v_p(l)$ is the driving noise at the pilot symbols' positions with dimension *PX1* such that

$$E[v(l)v^*(l+m)]=\begin{cases} V, & m=0 \\ 0, & m\neq 0 \end{cases} \tag{20}$$

and

$$V=(1-J_0(2\pi F_\mathrm{d} bT_\mathrm{s})WPW^\mathrm{T} \tag{21}$$

For first-order AR channel model, $\mathbf{F}$ is modeled as

$$\mathbf{F}=J_0(2\pi F_\mathrm{d} bT_\mathrm{s})WPW^\mathrm{T} \tag{22}$$

where b is a design parameter that determines the memory of the algorithm.

The input–output relation of the OFDM system is used to build the observation equation as

$$Y_p(l)=X_p(l)H_p(l)+W_p(l) \tag{23}$$

Now, it is possible to construct the Kalman channel estimator such that the gain, innovation process, channel impulse response and error covariance matrix are stated, respectively, as

$$K(l)=FP(l,l-1)X_p(l)^H[X_p(l)P(l,l-1)X_p(l)^H+\sigma^2 I]^{-1} \tag{24}$$

$$\alpha(l)=Y_p(l)-X_p(l)\dot{H}(l) \tag{25}$$

$$\dot{H}(l+1)=F\dot{H}(l)+K(l)\alpha(l) \tag{26}$$

$$P(l+1,l)=F[I-F^{-1}K(l)]P(l,l-1)F^H+V \tag{27}$$

where $X_p(l)$ is a *PXP* diagonal matrix with pilot symbols on its diagonal such that p is the number of pilot symbols and σ^2 is the variance of the AWGN.

5. THE LOW-COMPLEXITY KALMAN ESTIMATOR

A low-complexity Kalman estimator was implemented as described in [12]. The complexities of the Kalman estimator can be reduced by factorizing $P(l,l-1)$ using Eigenvalue decomposition such that

$$P(l,l-1)=UD(l)U^\mathrm{T} \tag{28}$$

where U is the unitary matrix whose columns are the Eigenvectors and $D(l)$ is a diagonal matrix with the Eigenvalues on its diagonal. Then it is possible to construct the Kalman estimator as

$$\alpha(l)=Y_p(l)-X_p(l)\dot{H}_p(l) \tag{29}$$

$$\dot{H}_p(l+1)=J_0(2\pi F_\mathrm{d} KT_\mathrm{s})[\dot{H}_p(l)+U.D(l)(D(l)+\sigma^2 I_m)]^{-1}\cdot U\cdot C_p\cdot\alpha(l) \tag{30}$$

To lower the complexity it is possible to update only the diagonal in D, the eigenvalues of $P(l,l-1)$, instead of updating the whole diagonal matrix D.

After the estimation process of the channel transfer function, an efficient interpolation technique is necessary, in order to estimate channel at data subcarriers, by using the channel information at pilot subcarriers. There are different types of interpolation schemes such as Linear Interpolation, Spline-cubic Interpolation, Low-pass interpolation, Second-order interpolation and Time domain interpolation [2].

6. SIMULATION RESULTS

Simulation results have been used to demonstrate the performance of the algorithm discussed above such that the low-complexity Kalman filter is used with comb-type pilot arrangement. The FFT size of the OFDM system used is 512, pilot ratio considered is $\frac{1}{8}$ and the Doppler frequency is 70 Hz, whereas guard band is $\frac{1}{32}$ from symbol duration, and Rayleigh fading channel is assumed. The performance of using the Kalman estimator with different interpolation techniques is illustrated in Figure 2. It is clear that Low-pass interpolation has better performance than others, since this method does the interpolation such that the mean-square error between the interpolated points and their ideal values is minimized. Moreover, changing the interpolation method has no effect on the performance for SNR values below 15 dB, but the difference becomes clearer for larger values of SNR.

The performance of comb-type pilot arrangements' channel estimation, using the Kalman estimator, is illustrated in Figure 3. It has been compared with LS estimator, which is particularly interesting since it is one of the most simple estimation methods. Low-pass interpolation method has been used as an interpolation technique. Obviously, the Kalman filter outperforms LS estimation especially for large values of SNR. However, the Kalman estimator is more complex.

The complexity of channel estimation, with comb-type pilot arrangements, based on the Kalman estimator can be reduced. Hence, the low-complexity Kalman estimator can be used as illustrated in Figure 4. It can be seen that the Kalman estimator performs better than the low-complexity Kalman method for SNR values greater than 15 dB. Hence, if the operating SNR is less that 15 dB then using the low-complexity Kalman estimator will be better in terms of complexity with no significant drop in performance.

One of the advantages of comb-type pilot arrangement, in channel estimation, is the ability to track the variation of the channel caused by the Doppler frequency. This is because every OFDM symbol has certain amount of pilots. In Figure 5, it is clear that although the Doppler

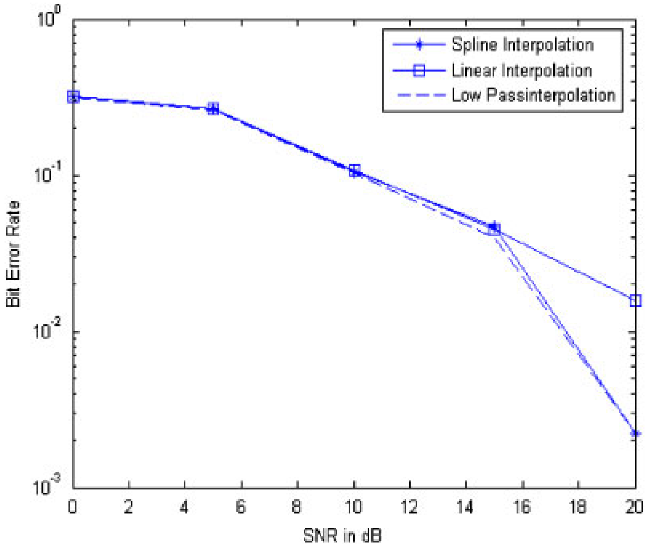

Figure 2. Performance of different interpolation methods using the Kalman estimator.

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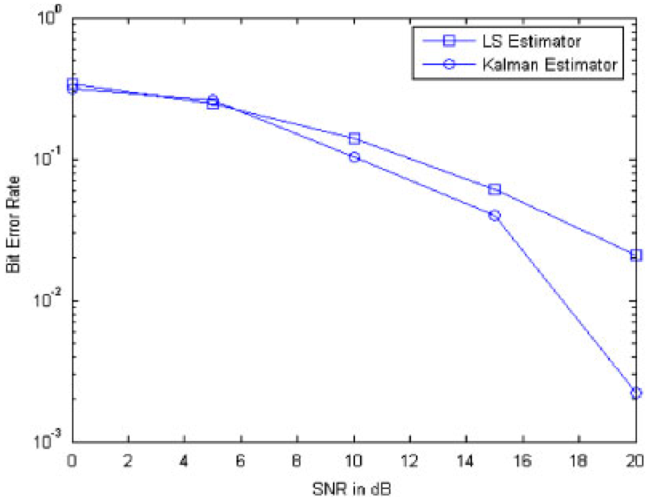

Figure 3. Performance of Kalman versus LS estimators.

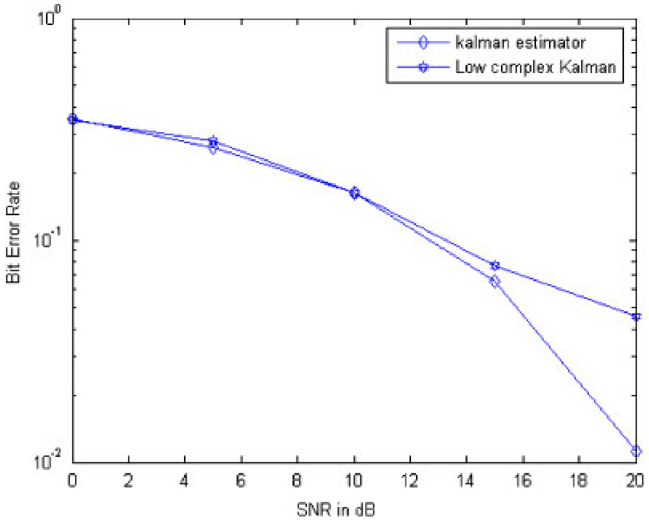

Figure 4. Comparison between the Kalman filter and the low-complexity Kalman.

frequency increases, the BER does not significantly increase at constant SNR, which means that system mobility has been enhanced. Accordingly, this method of pilot arrangements is interesting in OFDM system for WiMAX application. Doppler shift is the main reason of ICI and this will increase the noise level.

7. CONCLUSION

The Kalman filter has been used for the estimation of OFDM channels in a time-frequency-selective fading environment. An AR model of the channel has been assumed to model the fading process of a time-varying channel. The estimation is based on comb-type pilot frequencies' arrangement: channel coefficients that belong to the pilot subcarriers are estimated; then these estimates are interpolated. Low-pass interpolation method has given better performance compared with other methods since this technique interpolates such that the mean-square error between the interpolated points and their ideal values is minimized. Doppler frequency will destroy the orthogonality between

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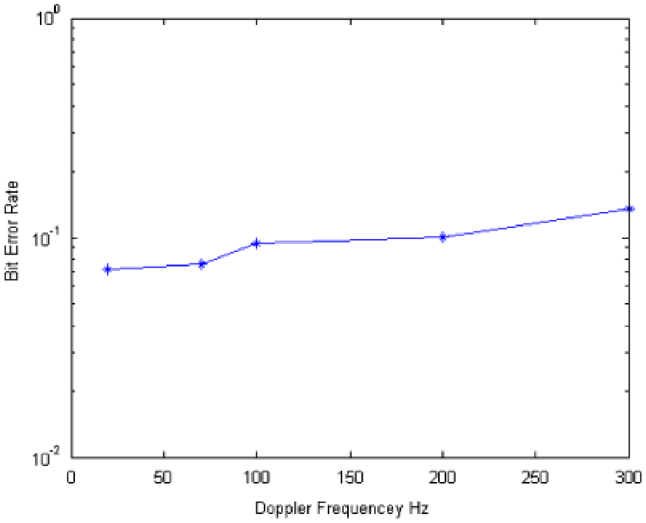

Figure 5. Doppler frequency effect for SNR = 15 dB.

subcarriers, and hence this will cause ICI, which degrades the performance of the estimation process. However, we have shown that utilizing the proposed low-complexity Kalman approach with comb-type pilot arrangement, the Doppler frequency shift is almost neglected. Kalman estimation outperforms LS estimation, but it increases the system complexity. It is a trade off between the system complexity and the performance required according to the desired SNR. But, Kalman and low-complexity Kalman estimators perform almost the same for SNR lower than 15 dB, which means that the choice of channel estimator is not that important in terms of symbol errors for low SNR. When choosing a channel estimating method for low SNR, the focus should instead be on how much information the estimating methods needs and also on how much the system mobility has been enhanced due to the Doppler effect reduction.

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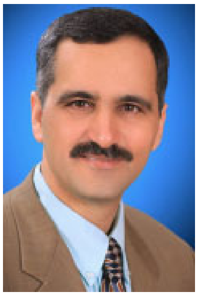

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