Fire Safety Journal 45 (2010) 211–220

Contents lists available at ScienceDirect

Fire Safety Journal

journal homepage: www.elsevier.com/locate/firesaf

Energy based time equivalent approach for evaluating fire resistance of reinforced concrete beams

V.K.R. Kodur*, P. Pakala, M.B. Dwaikat

Michigan State University, Civil and Environmental Engineering, 3580 Engineering Building, East Lansing, Michigan 48824-1226, USA

ARTICLE INFO

Article history:
Received 15 May 2009
Received in revised form
4 March 2010
Accepted 24 March 2010
Available online 22 April 2010

Keywords:
Fire resistance
Reinforced concrete beams
Time equivalent
Equal energy method

ABSTRACT

The current prescriptive approaches for evaluating fire resistance of reinforced concrete beams under standard fire exposure have a number of drawbacks and do not provide realistic performance assessment. As an alternative, time equivalent concept can relate the severity of design fire exposure to that of standard fire exposure. However, the current empirical formulae for evaluating such time equivalency are mainly derived for protected steel members and may not be applicable for reinforced concrete members.

This paper presents an energy-based time equivalency method for evaluating the fire resistance of reinforced concrete beams under design fire scenarios. The proposed method is based on the principle of equivalent energy, and estimates fire resistance based on the equivalency between standard and design fire exposures. The validity of the method is established by comparing the predictions from the proposed approach with those from existing methods (equivalent area method and empirical formulae) and with nonlinear finite element analysis. The applicability of the proposed approach to design situations is illustrated through a numerical example. It is shown that the proposed energy based method is capable of predicting equivalent fire resistance under design fire scenarios with an accuracy that is sufficient for practical purposes.

1. Introduction

Reinforced concrete (RC) structural systems are widely used in high rise buildings due to several advantages they offer over other construction materials. One such advantage is the high fire resistance, which eliminates the need of any external fire protection. Fire resistance is defined as the length of time over which a structural element retains structural integrity, and stability while displaying appropriate temperature transmission properties when exposed to fire conditions.

Fire resistance provisions for RC beams, in codes and standards, are derived based on results of standard fire resistance tests such as those specified in ASTM E119a [1] or ISO 834 [2]. Although standard fire resistance tests are helpful in assessing comparative performance of structural members, they do not take into account important factors such as realistic fire scenario, loading and failure criteria. The recent move towards performance based fire safety design emphasizes the need for evaluating performance of structural elements under realistic fire scenarios for rational and cost-effective design solutions [3,4]. Thus, current prescriptive based fire resistance approaches may not be applicable for evaluating fire resistance under performance based codes.

One of the main factors that influence the fire resistance of a RC beam is fire severity. Fire severity is defined as the destructive potential of a fire to which the structure is subjected [5]. In a prescriptive based approach, performance of an RC beam is evaluated in terms of standard fire exposure, which follows a predefined time–temperature curve as shown in Fig. 1. However, in a performance based approach, the fire resistance is to be evaluated under a realistic fire scenario that varies depending on the compartment characteristics such as fuel load, ventilation and lining materials.

The performance of structural members subjected to design fire exposures is often related to that of standard fire exposure by using time equivalent. The fire resistance equivalency, often referred to as time equivalent, can be defined as the time of exposure to standard fire that would result in the same fire severity as that of design fire exposure. By knowing the characteristics of a design fire, an equivalency can be established for the performance of an RC beam under a standard fire scenario. Numerous methods and empirical formulae for evaluating time equivalent of design fire scenarios have been developed. However, these methods and empirical formulae have a number of drawbacks since they do not account for all the factors governing fire response. Additionally they have not been validated for RC beams

* Corresponding author. Tel.: +1 517 353 5107; fax: +1 517 432 1827.
E-mail address: kodur@egr.msu.edu (V.K.R. Kodur).

0379-7112/$ - see front matter © 2010 Published by Elsevier Ltd.
doi:10.1016/j.firesaf.2010.03.002

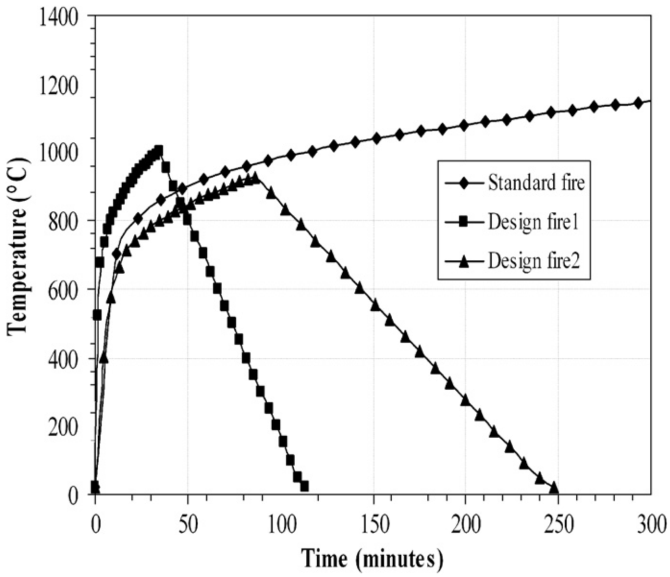

Fig. 1. Illustration of standard and representative design fire exposures.

over the full range of fire scenarios, and there is significant variation among various time equivalent approaches. Hence there is a need for an appropriate time equivalent method for establishing equivalency between standard fire exposure and a design fire exposure. The availability of such methods will lead to utilizing available fire resistance data under standard fire exposure and facilitate performance based design.

To overcome the above drawbacks, a new time equivalent approach is proposed for evaluating the fire resistance of RC beams under design fire scenarios. The proposed method is based on the principle of equivalent energy, and establishes equivalency between standard and design fire exposures. The proposed method is validated by comparing the predicted time equivalents with those obtained from nonlinear finite element (FE) analysis.

2. State-of-the-art

2.1. General

The fire resistance of RC beams is evaluated based on the results of standard fire test or through the use of calculation methods. The main drawback in these methods is that they are derived for standard fire exposure without any consideration to compartment characteristics such as fuel load and ventilation properties that influence the fire severity. Fig. 1 shows the time–temperature curve for ASTM E119 standard fire and two representative design fires (Fire 1 and Fire 2). Design fire, Fire 1 is a short hot fire whereas Fire 2 is a moderate fire. It can be seen from Fig. 1 that the temperature in the standard fire increases continuously without a decay phase whereas in the design fires there is a well defined decay phase. The presence of a decay phase in design fires significantly influences the fire response of structural members, and thus the current provisions in codes and standards may not provide a realistic assessment of fire performance of RC beams.

Recently, many countries are moving towards performance based fire design since this approach facilitates realistic and rational fire safety assessment under actual conditions present in a building. However, undertaking performance based fire design can present significant challenges since numerous fire scenarios are to be considered. Further, fire resistance tests or detailed finite element analysis are expensive and it is not possible to undertake fire resistance tests or simulations for all possible fire scenarios. One way of overcoming such a problem is by establishing an equivalency between the fire severity of a design fire and a standard fire exposure. Such an equivalency approach will eliminate the need for conducting fire tests and facilitate the establishment of fire resistance under any possible design fire scenario, provided the fire resistance is known for a standard fire exposure. In addition, existing data from standard fire resistance tests or detailed finite element analysis can be utilized for evaluating performance under design fire scenarios.

2.2. Equivalent fire severity (time equivalent)

Fire severity can be defined as the measure of destructive impact of a fire [5]. It depends on factors such as the amount of fuel load (occupancy type), size and location of openings, type of lining materials and compartment area. An RC beam should be designed such that its fire resistance, evaluated based on thermal, strength or deflection consideration, is higher than the severity of the fire to which the beam is exposed.

Equivalent fire severity, commonly referred to as time equivalent, can be used to evaluate the fire resistance of a structure by comparing the severity under a design fire exposure to that under a standard fire scenario. A number of methods and empirical formulae have been developed for evaluating the equivalent fire severity. These methods include equal area method, maximum temperature method, minimum load capacity method, and empirical formulae such as the CIB, Law and Eurocode formulae [6].

Equal area method establishes time equivalency by equating the area under the time–temperature curve of design fire scenario to that under the standard fire scenario. This method of comparing area of a standard fire to a realistic fire underestimates the heat transfer in a short hot fire and overestimates the heat transfer in a long cold fire, although both result in equal areas under two time–temperature curves [7]. Further this method has no rational basis and thus can give an incorrect assessment of design fires that deviate significantly from standard time–temperature curve. Despite these technical inadequacies, the equal area concept is often used to correct the results from a standard fire-resistance test when the time–temperature curve in a furnace does not follow the limits described in the standard [1].

Maximum temperature method computes time equivalency in a protected steel member by mapping the peak temperatures resulting from design fire exposure to the corresponding temperature under standard fire exposure. This method was derived based on temperature measurements in protected steel structural members exposed to fire and may not be fully applicable to RC members. In addition, it may not be accurate if the maximum temperatures used for computing the time equivalent are much higher or lower than those which would cause failure in a particular member [5]. Due to lack of other methods, maximum temperature method is often applied to evaluate time equivalency for RC beams by using rebar temperatures. However, computing rebar temperature requires detailed thermal analysis of the beam section which may involve significant computational effort. Therefore this method cannot be easily applied in design situations. Furthermore, the method has not yet been validated for RC beams. Also due to limited validation the method may not be applicable to wide range of fire scenarios, encountered in practice.

Minimum load capacity establishes time equivalency for a structural member by comparing the minimum load capacity reached under design fire scenario to the corresponding load capacity attained under standard fire exposure. This method of establishing time equivalent captures various critical factors, such as fire scenario, beam characteristics, high temperature material properties, load level and support conditions. Thus, it generally results in a better estimate of time equivalency as compared to equal area or maximum temperature methods. However, it does not consider deflections that may be governing failure under some conditions. In addition, to determine strength under design and standard fire scenarios, detailed strength analysis is required which can take significant computational effort and this limits the application of this method in design situations.

Similar to minimum load capacity method, maximum deflection method computes time equivalency by comparing the maximum deflection obtained under design and standard fire scenarios. This method takes into account deflection failure limit state, which is often the governing criterion in RC beams and this method is more reliable for estimating time equivalency. Failure in an RC beam can occur if the integrity cannot be maintained under large deflections. Also large deflections can lead to the development of wider cracks that leave the reinforcement directly exposed to fire and can produce sudden failure in the beam.

With the exception of the equal area method, all three methods (maximum temperature method, minimum load capacity method and maximum deflection method) require detailed finite element analysis and thus cannot be easily applied in design situations.

Apart from the four methods described above several empirical formulae, such as CIB, Law and Eurocode formulae are derived based on the maximum temperature of protected steel members, is available for computing time equivalent. Full details of these formulae, including limitation of its applicability is described elsewhere [6].

2.3. Comparison of time equivalent approaches

Though the above empirical formulae are useful in computing time equivalency, the results from these formulae are not consistent. Also, the time equivalent computed by these formulae shows a significant variation even for similar fire exposure. Further, these formulae are derived for protected steel members and may not be fully applicable for RC members.

To illustrate the variation in time equivalency predicted by various methods and empirical formulae, the above formulae was applied for evaluating the equivalent fire resistance of an RC beam. The RC beam used in the case study is a simply supported beam of 6 m span length and is made of concrete with a compressive strength of 30 MPa and reinforced with steel rebars having yield strength of 400 MPa. This beam is subjected to a representative design fire FS5 (details of FS5 are presented in Table 2) and the time equivalency is calculated based on equal area method and empirical formulae discussed above. CIB formula gives a time equivalent of 174 min whereas Law and Eurocode formulae predict a time equivalent to be 132 and 155 min respectively. Time equivalent predicted by equal area method is about 232 min. It can be clearly seen that there is a wide variation in time equivalent predicted by various methods. This can be attributed to the fact that the above methods and the empirical formulae have been derived based on the maximum temperature in protected steel members and may not be applicable to RC members.

2.4. Summary

The above review clearly illustrates that the current approaches of evaluating fire resistance in RC beams are based on prescriptive methods. These prescriptive approaches do not take into consideration the type of fire exposure which has significant influence on fire resistance of structural members. There are few time equivalent methods to establish equivalency between standard and real fire exposure. But many of these methods have limitations and may not be fully applicable to RC members. There is a large amount of data on fire resistance of RC beams under standard fire exposure, and this can be used for establishing equivalency under design fire exposure if a reliable time equivalent approach is available. Thus, development of a reliable time equivalent approach can facilitate evaluation of fire resistance under design fire scenarios.

3. Numerical studies

3.1. General

The development of a reliable time equivalent approach for RC beams requires large set of data under standard and design fire exposures. Such data can be generated through numerical simulations on RC beams under various fire scenarios. For the analysis, a macroscopic finite element (FE) based computer program was selected. The advantage of utilizing the FE program is that various fire, and support conditions can be accounted for evaluating the fire response of RC beams.

Different type of RC beams were analyzed under 18 fire scenarios and four different support conditions resulting in 72 beam-fire combinations. Results from FE analysis was used to establish time equivalent of each beam under a given design fire exposure. This predicted time equivalent is considered to be reliable since finite element analysis accounts for various factors such as support conditions, high temperature material properties and fire scenario that influence fire resistance of RC beams. Details on the computer program, the analysis and the analyzed beams are discussed in the following section.

3.2. Macroscopic finite element model

The numerical studies for establishing fire resistance data on RC beams was carried out using a macroscopic finite element model [8]. The RC beam is discretized into a number of segments along its length and the mid-section of each segment is assumed to represent the behavior of the whole segment. The mid-section is further divided into elements forming two dimensional mesh. The boundaries restraining the beam are idealized as a spring of stiffness (k) as shown in Fig. 2.

The fire resistance analysis is carried out at various time steps, till failure occurs in the beam. At each time step the fire temperature is first evaluated for a specified fire scenario. Then a heat transfer analysis is carried out using FE approach to compute the temperature distribution across the cross section of each segment. These cross-sectional temperatures serve as input to subsequent strength analysis.

The fire-induced axial restraint force is computed by satisfying compatibility and equilibrium criteria along the span of the beam. At each time step the computed temperature and fire-induced axial restraint force are utilized to generate moment curvature relationships in various segments. The generated moment curvature relationships are used to evaluate the stiffness of various segments and to undertake a nonlinear structural analysis to predict the response of the RC beam at each time step. The model accounts for creep and transient strain components, high temperature material properties, geometric and material nonlinearity and material softening.

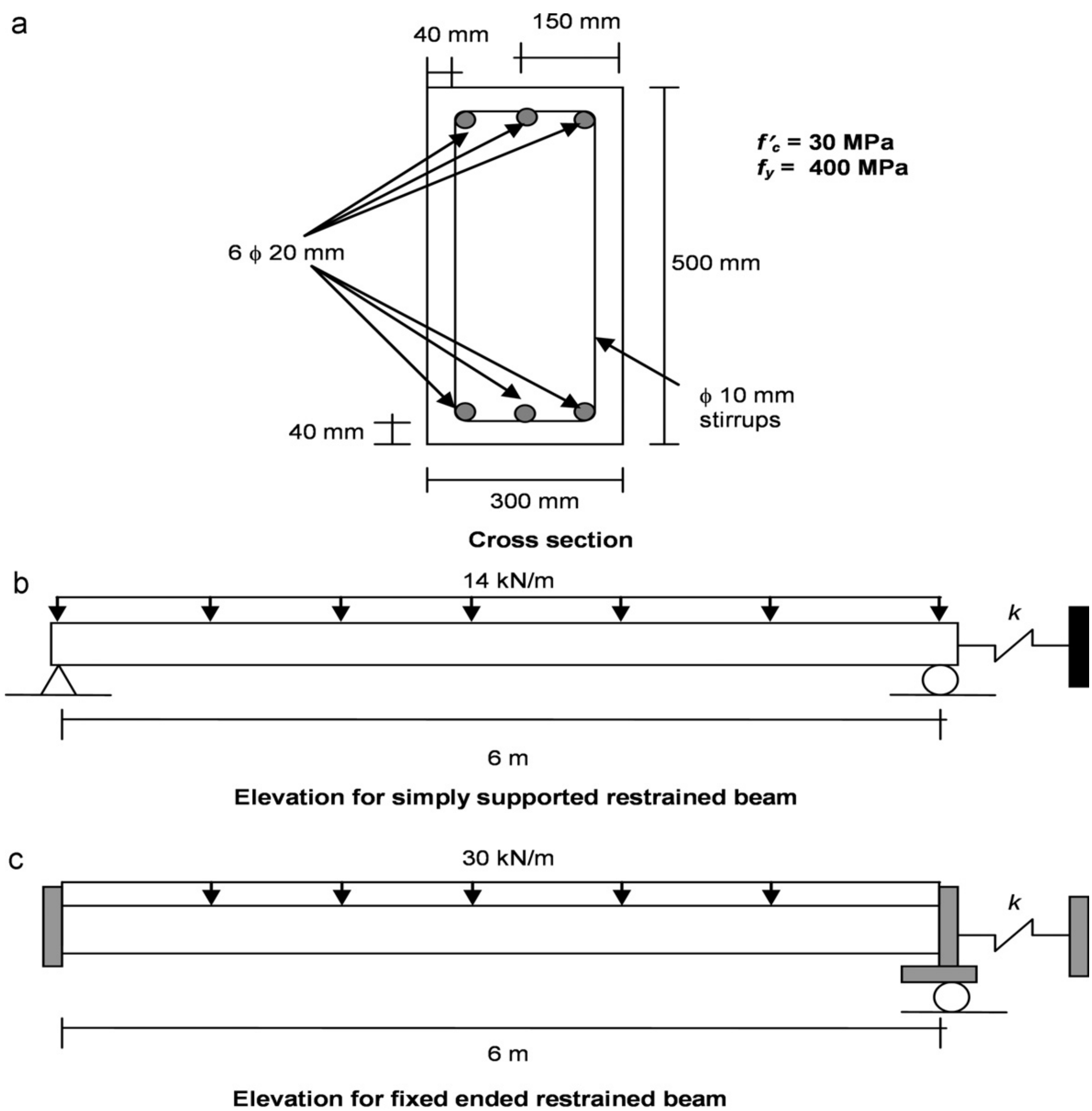

Fig. 2. Cross section and elevation of RC beam used in the analysis.

The model is capable of undertaking fire resistance analysis for beams exposed to any given fire scenario (time temperatures). The computer model also generates temperatures, stresses, strains, moments and deflections at each time step. These output parameters are used to check the failure of the beam at each time step. The analysis terminates when failure of the beam occurs under any specified limiting criterion. Full details of the numerical procedure, including derivation of appropriate equations, are presented by Kodur and Dwaikat [8].

3.3. Design parameters

For generating time equivalent data, four RC beams (each with different support conditions) were analyzed. The four beams had a simply supported, axially restrained, rotationally restrained and both axially and rotationally restrained end conditions. Details of the beam and boundary conditions are illustrated in Fig. 2. Two values of axial restraint stiffness, namely 0 and 20 kN/mm, are selected for the analyzed beams.

All the beams are of rectangular cross section ($300 \times 500\ mm^2$) and have a span length of 6 m as shown in Fig. 2. The beams are assumed to be made of concrete with a compressive strength of 30 MPa and reinforced with steel rebars having yield strength of 400 MPa. The applied load on simply supported and axially restrained beams was 14 kN/m (equivalent to a load ratio of 40%), while the corresponding applied load on rotationally restrained, rotational and axially restrained beams was 30 kN/m (equivalent to a load ratio of 40%). Load ratio is defined as the ratio of expected loads on the beam during a fire to the loads that would cause collapse of beam at room temperatures. The fire resistance analysis was carried out in 2.5 min time increments for a maximum fire exposure time of 8 h. Data from fire resistance analysis was used to derive time equivalent for the seventeen design fires with respect to that of standard fire exposure.

3.4. Fire exposure

In order to generate data for applying the equivalent fire severity concept, the beams were analyzed under one standard fire [1,9] and seventeen design fire scenarios namely FS1 through FS17. ASTM E119 fire represents the fire scenario used in standard fire resistance tests and is similar to other standard fire scenarios specified in standards such as ISO 834 [2] standard fire. Fig. 3 shows the time–temperature curves for the standard and various design fires (FS1 through FS17) used in the analysis. The design fires are selected to cover wide range of compartment characteristics and fuel loads that are encountered in different types of occupancies (buildings). The parametric fire time–temperature curve[10] proposed in Eurocode 1 [11] and the recent modifications suggested by Feasey and Buchanan [12] are implemented to arrive at different design fire scenarios. According to Eurocode 1, a design fire consists of a growth phase and a decay phase. Feasey and Buchanan [12] showed that both the growth and decay phases of the fire are influenced by compartment properties such as the fuel load, ventilation opening and wall linings. These design fires are assumed to occur in a room of dimension $6 \times 4 \times 3$ m^3. Values of fuel loads ranging from 400 to 1600 MJ/m^2 of floor area are used. Opening dimensions are assumed such that the ventilation factor is between 0.02–0.04 m$^{0.5}$. In order to account for the realistic nature of lining material such as gypsum board, concrete and composite construction material, values of thermal inertia (b) (given by Eq. (9)) are assumed to vary from 488 to 1900 Ws$^{0.5}$/m^{2}K. The different compartment characteristics utilized to establish the design fire scenarios are presented in Table 1.

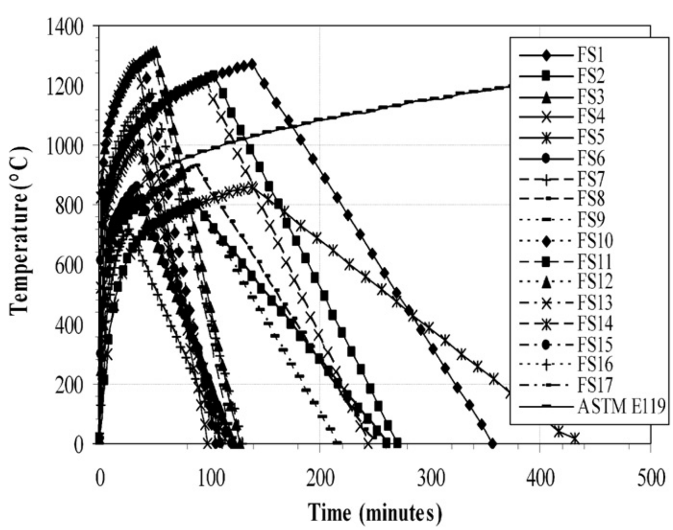

Fig. 3. Time–temperature curves for design and ASTM E119 standard fire exposure.

3.5. Evaluating time equivalent

The time equivalent for the analyzed beams was evaluated using the maximum deflection method and the results from FE analysis described above. The maximum deflection method is selected (over minimum load capacity method) since the failure of an RC beam under fire exposure is generally governed by deflection failure criteria as discussed above. It should be noted that the fire resistance based on deflection criteria is very close to that of strength criterion.

3.6. Results

The analysis of four types of RC beams under one standard fire and seventeen design fires produced seventy two time–deflection curves. These time–deflection curves are utilized to compute the time equivalent for each beam-fire combination using the maximum deflection method. The time equivalent for each combination is also evaluated using the empirical formulae and the equal area method. A comparison of estimated time equivalent values based on various methods with that predicted by the maximum deflection method for the 72 beam-fire combinations are shown in Fig. 4. The conservative and

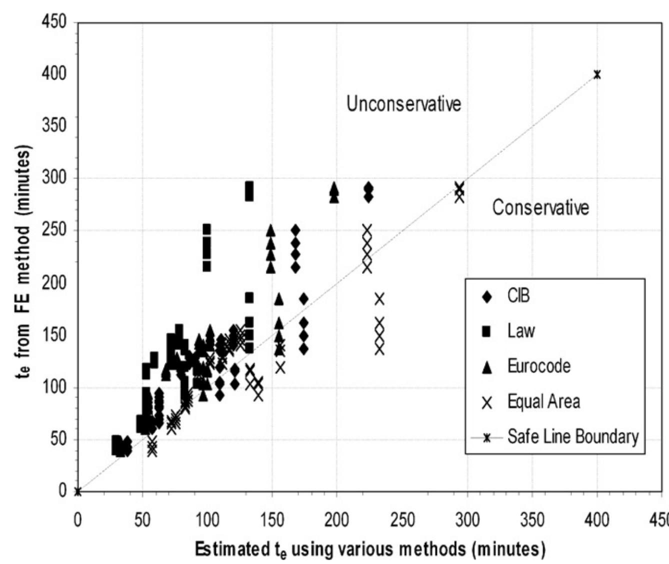

Fig. 4. Comparison of time equivalent computed based on FE analysis with that of other methods.

Table 1
Compartment characteristics used for arriving at different design fire scenarios.

Fire scenario	Fuel load (MJ/m^2 floor area)	Ventilation factor (m$^{0.5}$)	Thermal capacity (Ws$^{0.5}$/m^{2}K)
FS1	1600	0.02	488
FS 2	1200	0.02	488
FS 3	1200	0.04	488
FS 4	800	0.04	488
FS 5	1600	0.02	1900
FS 6	800	0.04	1900
FS 7	400	0.026	1900
FS 8	1100	0.04	488
FS 9	1300	0.03	1900
FS 10	900	0.04	488
FS 11	1000	0.02	1900
FS 12	700	0.035	1900
FS 13	1100	0.02	488
FS 14	800	0.04	1200
FS 15	800	0.04	1000
FS 16	1100	0.04	800
FS 17	1000	0.02	1200

unconservative regions for the data points are also shown in Fig. 4. The comparative study did not include the minimum load capacity and the maximum temperature methods since these methods may not provide realistic fire resistance values and also because these methods require detailed finite element analysis which limits their use for design purposes. The time equivalency predicted by maximum deflection method is considered to be the most reliable value for time equivalency.

It can be seen from Fig. 4, that there is significant variation in the time equivalent (t_e) values predicted by various methods and empirical formulae. Almost all the time equivalent values predicted by CIB formula are unconservative and the variation in the predicted t_e values increases as the severity of the design fire increases (as the time equivalent value increase).

Similar trends can be seen in Fig. 4 for the time equivalent values computed using the Law formula. The time equivalent values predicted by the Law formula are highly unconservative as compared to those predicted by CIB formula. Similar to CIB formula, the time equivalent values predicted by the Law formula are more scattered for severe design fire scenarios. Time equivalent values predicted by Eurocode formula are also unconservative and follow similar trend as that of CIB and Law formula. In general, the time equivalent values predicted by empirical formulae are unconservative and the accuracy of prediction decreases as the fire severity increases.

As can be seen in Fig. 4, the time equivalent values predicted by the equal area method have less variation than those predicted by the empirical formulae. However, similar to different empirical formulae, the variation in the time equivalent values predicted by equal area method becomes significant for severe design fire scenarios. Also, equal area method gives unconservative predictions for almost half of the time equivalent values computed in this study. This clearly indicates that the time equivalent computed based on the equal area method may not be conservative under many fire scenarios.

In summary, the time equivalent values predicted by equal area method and empirical formulae are generally unconservative and have significant variation. Thus, the current time equivalent methods do not yield reliable fire resistance predictions for RC beams. To overcome this drawback, a reliable and conservative semi-empirical approach for establishing time equivalency of RC beams is developed and is presented in the following section.

4. Development of time equivalent methodology

4.1. General principle

The proposed methodology for establishing time equivalency between standard and design fire scenarios is based on equivalent energy concept. The energy based concept is better suited, than equal area or maximum temperature approach, for establishing equivalency since equal energy concept relates the fire severity, and thus resulting fire resistance, to the amount of energy transferred to the beam. Accordingly, two fires will have the same fire severity if they transfer same amount of energy to an RC beam. The amount of energy transferred to an RC beam exposed to fire is related to the heat flux on the fire exposed boundaries of the beam, which involves heat transfer through convection and radiation. The convection and radiation heat flux on the boundary of an RC beam exposed to fire can be given by the following two formulae respectively: [5]

$$q_c = h_c(T_f - T_c) \tag{1}$$

$$q_r = \sigma\varepsilon({T_f}^4 - {T_c}^4) \tag{2}$$

where q_c=convective heat flux (W/m^2), q_r=radiative heat flux (W/m^2), h_c=convective heat transfer coefficient (W/m^{2}K), T_f=fire temperature (θ or K), T_c=temperature on surface of boundary (θ or K), σ=Stefan–Boltzmann constant (5.67×10^{-8} W/m^{2}K^4) and ε=emissivity.

Since the temperature on the exposed surface (T_c) is generally close to the fire temperature (T_f), the radiation heat flux can be approximated as follows:

$$\begin{aligned} q_r &= \sigma\varepsilon({T_f}^4 - {T_c}^4) \\ &= \sigma\varepsilon(T_f^2 + T_c^2)(T_f^2 - T_c^2) \\ &= \sigma\varepsilon(T_f^2 + T_c^2)(T_f + T_c)(T_f - T_c) \\ &\approx \sigma\varepsilon(T_f^2 + T_f^2)(T_f + T_f)(T_f - T_c) \\ &\approx 4\sigma\varepsilon {T_f}^3(T_f - T_c) \end{aligned} \tag{3}$$

Thus, the total heat flux can be written as:

$$q = q_c + q_r \approx \; h_c(T_f - T_c) \; + \; 4\sigma\varepsilon {T_f}^3(T_f - T_c) \tag{4}$$

Assuming $T_f - T_c = \alpha\, T_f$, (where α is a constant), Eq. (4) can be written as

$$q \approx \alpha(4\sigma\varepsilon {T_f}^4 + h_c T_f) \tag{5}$$

Thus, the amount of energy transfer to an RC beam under fire exposure can be approximated by the following formula:

$$E = \int qA\,dt \approx \int A\alpha(4\sigma\varepsilon {T_f}^4 + h_c T_f)\,dt \tag{6}$$

where A=area of boundary exposed to fire and E=total energy

Since both A and α are assumed to be constant, they can be taken outside the integral in Eq. (6) and hence

$$E = \alpha A \int (4\sigma\varepsilon {T_f}^4 + h_c T_f)\,dt \tag{7}$$

or

$$E = \alpha A \times \text{Area under heat flux curve}\left(\frac{q}{\alpha}\right) \tag{8}$$

The term 'E' represents the energy bound by the time-temperature curve of given fire exposure. This energy based approach can only be applied if the temperature of the compartment can be assumed as a single temperature and the convective heat transfer coefficient and emissivity have to be homogeneous along the structural element. In the current study, an emissivity (ε) value of 0.5 and convective heat transfer coefficient (h_c) of 25 W/m^{2}K are used.

Thus using the equivalent energy principle, a design fire will have the same severity as that of the standard fire if

$$E_s = E_d \tag{9}$$

where E_s=total energy under the heat flux (q/α) curve of the standard fire, and E_d=total energy under the heat flux (q/α) curve of the design fire.

Consequently, the equivalent time can be computed by equating the total area under the heat flux (q/α) curve for the design fire with the area under the heat flux (q/α) curve for the standard fire as shown in Fig. 5 (for the standard and design fires). To arrive at equivalency, first the total area under the heat flux curve for the design fire (area B in Fig. 5) is computed. The area under the heat flux of a standard fire (area A in Fig. 5) is computed at various time steps. The time at which area A (which varies as a function of time) equals area B is the time equivalent of the design fire. The steps involved in computing the time equivalent using the proposed energy method are illustrated by solving a numerical example presented in Appendix A.

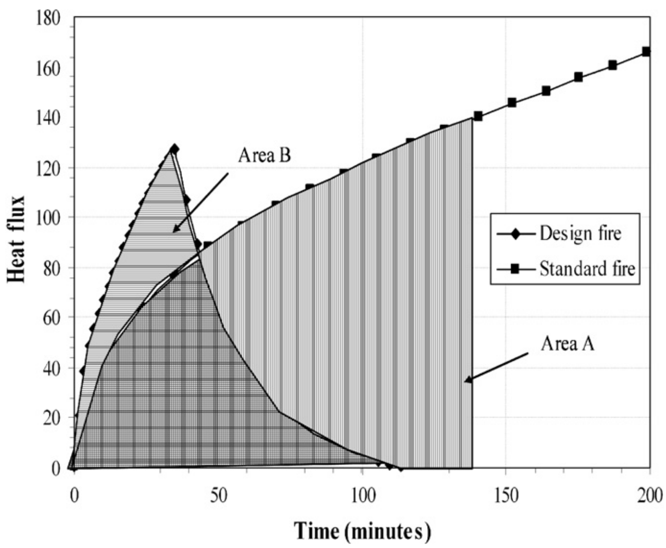

Fig. 5. Equivalent energy concept for standard and design fire.

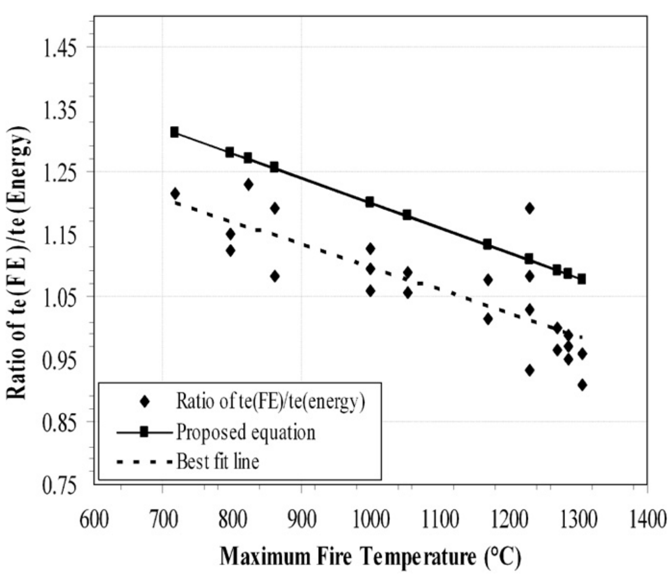

Fig. 6. Variation of $t_{e(FE)}/t_{e(energy)}$ with maximum fire temperature.

4.2. Calibration

To improve the accuracy of the time equivalent predictions from the proposed method, estimated time equivalency values have been calibrated against time equivalent values obtained from finite element analysis. It has been found that there exists a correlation between the ratio of the two time equivalent values, values predicted by the FE method and the equal energy method ($t_{e(FE)}/t_{e(energy)}$), and the maximum temperature of design fire as shown in Fig. 6. It can be seen from Fig. 6 that the ratio of time equivalent predicted by the FE method to that predicted by equal energy method decreases with increase in the maximum temperature of design fire. This can be attributed to the fact that the damage of beam depends not only on energy transferred from fire but also on other factors such as temperature distribution and thermal gradients. These thermal gradients are large in case of fires with high maximum temperatures.

The data generated from FE analysis was randomly divided into two sets. The first set was used for the calibration of the method and the second set was used to validate the method. Almost half of the beam-fire combinations have been selected at random and the ratio of time equivalents obtained by FE and energy method are plotted against the maximum temperature reached in respective fire scenarios as can be seen from Fig. 6. Least sum of square of errors analysis is carried out to obtain a best fit and a conservative line for that correlation and is shown in Fig. 6. Accordingly, the equation of conservative line for predicting the ratio between the two time equivalents ($t_{e(FE)}/t_{e(energy)}$) is given as

$$\frac{t_{e(FE)}}{t_{e(energy)}} = 1.6 - 0.0004 \times T_{max} \tag{10}$$

where $t_{e(FE)}$=time equivalent computed from maximum deflection method (or FE analysis), $t_{e(energy)}$=time equivalent computed from equivalent energy method and

T_{max}=maximum temperature of design fire

Thus, the actual time equivalent of a design fire can be estimated by the following equation

$$t_{e(FE)} = (1.6 - 0.0004 \times T_{max})t_{e(energy)} \tag{11}$$

4.3. Validation

To illustrate the validity of the proposed method, the time equivalent values computed based on the energy method are compared with the time equivalent values obtained from the FE analysis. A summary of computed t_e values are presented in Table 2. Fig. 7 shows variation between the time equivalent values predicted by various methods, empirical formulae, FE analysis and proposed equal energy method. It can be seen from Fig. 7 that almost all the time equivalent values predicted by the equal energy method are on the conservative side throughout the range of fire scenarios considered. Time equivalents predicted by equal area method shows less scatter as compared to other conventional methods and empirical formulae. The figure also shows the time equivalent computed based on equal energy method have less variation as compared to other methods. Thus, equal energy method can be considered to be reliable method for estimation of time equivalent of design fires.

5. Design Implications

Current approach of evaluating fire resistance of RC beams is mostly based on standard fire exposure. The recent move towards performance based fire design require the development of rational approaches for evaluating fire resistance based on realistic fire conditions.

The currently available time equivalent methods have a number of drawbacks and are not directly applicable for evaluating equivalent fire resistance of RC beams. The proposed approach, based on equivalent energy principles, represents a better measure of fire severity (equivalency) than conventional equivalency methods such as equal area and maximum temperature concepts. Therefore time equivalent predictions based on energy approach provides better estimate of fire resistance. Further the simplicity of the proposed relationship makes it attractive for incorporating in codes and standards.

The proposed equal energy method can be used to estimate the time equivalent under a given design fire scenario if the fire resistance under a standard fire exposure is known. For example, if fire resistance (based on test or analysis) for an RC beam under standard fire scenario is known then energy based approach can be applied to develop fire resistance of that beam for any possible design fire scenario. The first step is to compute time equivalent of design fire scenario which can be done through simple

Table 2
Summary of computed time equivalent values from various methods.

Design fire	Duration of burning period (min.)	Maximum temperature (θ)	Time equivalent (min.)					
			Equal Energy method	Equal area method	Finite element analysis			
					Simply supported	Axially restrained	Rotationally restrained	Axially and rotationally restrained
FS1	139	1270	304	294	*	290	292	282
FS2	104	1230	231	223	250	237	227	215
FS3	52	1314	162	125	150	155	145	140
FS4	35	1270	118	93	118	117	115	112
FS5	139	861	150	232	185	150	162	137
FS6	35	861	59	75	73	70	70	65
FS7	27	717	39	57	47	42	40	40
FS8	48	1306	151	117	142	145	137	135
FS9	75	890	99	132	117	115	115	102
FS10	39	1285	129	101	125	127	122	122
FS11	87	798	91	139	105	102	105	92
FS12	35	824	55	72	67	65	65	60
FS13	104	1229	210	223	250	237	227	215
FS14	35	1000	76	82	87	82	85	80
FS15	35	1054	83	85	95	90	90	87
FS16	48	1170	121	111	130	130	127	122
FS17	87	928	118	156	140	135	135	120

*Not applicable as failure occurred before beam reached maximum deflection.

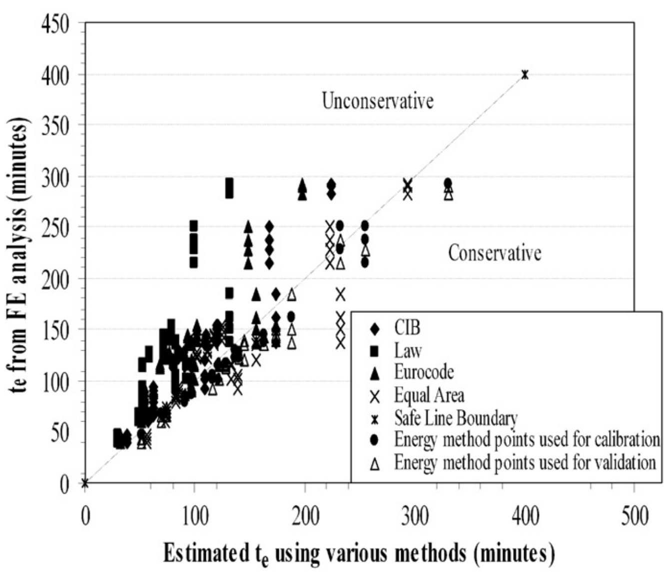

Fig. 7. Comparison of time equivalent from equal energy method with various methods.

spreadsheet calculations as illustrated in Appendix A. The second step is to apply Eq. (11) to obtain conservative estimate of actual time equivalent (revised) under that design fire. Finally a comparison is to be carried out between the revised time equivalent and fire resistance under standard fire to check if the RC beam meets the required fire resistance rating. Failure of the beam is said to occur only if the revised time equivalent is greater than the fire resistance under standard fire exposure. Thus the proposed equal energy method can be used to arrive at a conservative estimate of actual time equivalent (using Eq. (11)) without the need for detailed FE analysis. Hence the proposed approach can be used for rational fire design of RC members under performance based design environment.

6. Conclusions

- Current methods and empirical formulae for evaluating time equivalency of structural members under design fire exposures have large variation and are generally unconservative.
- The proposed energy based approach establishes time equivalency based on total energy transferred to an RC beam from a design fire to that of a standard fire exposure. Thus the method has stronger scientific basis than equal area or maximum temperature method.
- The energy based approach provides better estimate of time equivalent for RC beams than current approaches.
- The proposed energy based time equivalent approach is capable of predicting time equivalent of design fires with an accuracy that is sufficient for design purposes.

Acknowledgements

The research, presented in this paper, is primarily supported by the Michigan State University, through the Strategic Partnership Grant Award (SPG 71-4434) to the first author.

Appendix A. Step-by-step procedure

To illustrate the applicability of energy based method, time equivalent for design fires can be evaluated using a spreadsheet. The three main steps are:

- Computing the total energy of design fire.
- Computing cumulative energy of standard fire and
- Finding the time equivalent.

To further illustrate the step-by-step procedure, a case study is presented here. In this case study, the time equivalent of an RC beam is evaluated using the proposed method. The properties of RC beam are explained in Section 3.3. The beam is assumed to be exposed to design fire FS1 with a total duration of 355 min, shown

Table A1
Step – by – step calculations for evaluating time equivalent of fire scenario1 (FS1) by equal energy method.

Time step	Time (min)	Design fire temperature (θ)	Heat flux (q/α) (W/m^2)		Energy (Joules) = Cumulative area under heat flux curve		Difference in total energy of design fire and cumulative energy of standard fire
			Design fire	Standard fire	Design fire	Standard fire	
1	$t_1=0$	$T_{f1}=20$	Heat flux at time step t_1 (HFt$_1$_DF) calculated using Eq. (5)=8161	Heat flux at time step t_1 (HFt$_1$_SF) calculated using Eq. (5)=8161	$ED_1=0$	$ES_1=0$	ES_1-ED_n =(1782332 − 0) = 1782332
2	$t_2=t_1+\Delta t$ = 0.5	$T_{f2}=319$	Heat flux at time step t_2 (HFt$_2$_DF) calculated using Eq. (5)=28730	Heat flux at time step t_2 (HFt$_2$_SF) calculated using Eq. (5)=22094	ED_2=0.5 × (t2 − t1) × (HFt1_DF +HFt2_DF)+ED_1 = 154	ES_2=0.5 × (t2 − t1) × (HFt1_SF +HFt2_SF)+ES_1 = 127	ES_2-ED_n =(1782332−127) = 1782205
..							
i	$t_i=303$	$T_{fi}=303$	Heat flux at time step t_i (HFt$_i$_DF) calculated using Eq. (5)=28162	Heat flux at time step t_i(HFt$_i$_SF) calculated using Eq. (5)=506521	ED_i=0.5 × (t_i-t_{i-1}) × (HFt$_i$_DF+ HFt$_{i-1}$_DF)+ED_{i-1}=1768628	ES_i=0.5 × (t_i-t_{i-1}) × (HFt$_i$_SF +HFt$_{i-1}$_SF)+ES_{i-1}=1782530	ES_i-ED_n =(1782332 − 1782530) ≈ 0
..							
$n-1$	$t_{n-1}=350$	$T_{fn-1}=20$	Heat flux at time step t_{n-1} (HFt$_{n-1}$_DF) calculated using Eq. (5)=8951	Heat flux at time step t_{n-1} (HFt$_{n-1}$_SF) calculated using Eq. (5)=544264	ED_{n-1}=0.5 × ($t_{n-1}-t_{n-2}$) × (HFt$_{n-2}$_DF+HFt$_{n-1}$_DF) +ED_{n-2} = 1781634	ES_{n-1}=0.5 × ($t_{n-1}-t_{n-2}$) × (HFt$_{n-2}$_SF+HFt$_{n-1}$_SF) +ES_{n-2}=2220220	$ES_{n-1}-ED_n$ =(1782332 − 2220220) = −437888
n	$t_n=355$	$T_{fn}=20$	Heat flux at time step t_n (HFt$_n$_DF) calculated using Eq. (5)=8161	Heat flux at time step t_n (HFt$_n$_SF) calculated using Eq. (5)=548474	ED_n=0.5 × (t_n-t_{n-1}) × (HFt$_n$_DF +HFt$_{n-1}$_DF)+ED_{n-1}=1782332	ES_n=0.5 × (t_n-t_{n-1}) × (HFt$_n$_SF+HFt$_{n-1}$_SF) +ES_{n-1} =2266281	ES_n-ED_n =(1782332–2266281) = −483949
			Total Energy of design fire		ED_n=1782332		

in Fig. 3, and the time equivalent is evaluated as follows and the calculations are illustrated in Table A1.

Computing the total energy of design fire can be divided into the following sub-steps:

- The total duration of fire (355 min) is divided into half minute (8.333×10^{-3} h) time increments (Δt).
- Starting with an initial time of zero, fire temperature is calculated at each time step using the time–temperature relationships specified in standards.
- The fire temperature computed above is used to compute the heal flux (q/α) using Eq. (5).
- At each time step, area under the heat flux curve (energy) is calculated using Trapezoidal rule. For example, at time step 2, the energy of fire is computed to be 154 Joules.
- The computed values of energy at each time step are summed up to give the total energy of design fire. Thus a total energy of 1 782 332 Joules is obtained for this case study.

Computing the cumulative energy of standard fire can be done through the following sub-steps:

- The standard fire used in this case study is ASTM E119 with a maximum duration of six hours (360 min).
- The total duration of 360 min is divided into half minute (8.333×10^{-3} h) time increments.
- At each time step, the temperature of standard fire is computed using the approximate time–temperature relationship provided by Lie (1995).
- Using this value of fire temperature, heat flux at each time step is computed using Eq. (5) followed by the computation of area under heat flux curve (energy), using Trapezoidal rule.
- At each time step compute the cumulative area under heat flux curve (energy) of the standard fire exposure which is equal to the energy at the current time step plus the sum of energies till the previous time step.
- At each time step, the difference between the total energy of design fire and cumulative energy of standard fire is computed.

The difference between two energy values is approximately zero at a time of 303 min. This time value is defined as the time equivalent.

In summary, the proposed method predicts a time equivalent value of 303 min with respect to ASTM E119 standard fire exposure for the RC beam described in the case study subjected to design fire scenario FS1.

The same beam is analyzed (using the FE analysis) under ASTM E119 standard fire exposure and the fire resistance of the beam is found to be equal to 227 min. Since the computed time equivalent is more than the fire resistance (failure time) of the beam, the beam cannot survive complete burnout of design fire FS1 without failure. In summary, failure of the beam happens only if the time equivalent obtained is greater than the fire resistance of the beam.

Conservative estimate of the actual time equivalent for the design fire FS1 can be computed from Eq. (11), using the time equivalent obtained from equal energy method without the need for detailed FE analysis. For design fire FS1, maximum fire temperature is equal to 1270 θ. Using Eq.(11) gives a time equivalent of 330 min. Thus the RC beam should be designed to sustain a standard fire for a minimum duration of 330 min. Hence the actual time equivalent value of design fire can be calculated using simple spreadsheet calculations without the necessity of detailed FE analysis.

References

[1] ASTM, Standard methods of fire test of building construction and materials, Test Method E119a -08, American Society for Testing and Materials, West Conshohocken, PA, 2008.

[2] ISO 834-1975, Fire resistance tests- elements of building construction, Int. Org. Stand. (1975).
[3] V.K.R. Kodur, M.B. Dwaikat, Performance-based fire safety design of reinforced concrete beams, J Fire Prot. Eng. 17 (4) (2007) 293–320.
[4] B.J. Meacham, R.L.P Custer, Performance-based fire safety engineering: An Introduction of basic concepts, J. Fire Prot. Eng. 7 (2) (1992) 35–54.
[5] A.H. Buchanan, in: Structural Design for Fire Safety, John Wiley and Sons Ltd., Chichester, England, 2002.
[6] P. Pakala, Energy based equivalent approach for evaluating fire resistance of reinforced concrete beams. Master's Thesis, Department of Civil and Environmental Engineering, Michigan state university, East Lansing, Mi, USA, 2009.
[7] J.F. Nyman, Equivalent fire resistance ratings of construction elements exposed to realistic fires. Master's research report, Department of Civil Engineering, University of Canterbury, New Zealand, May 2002.
[8] V.K.R. Kodur, M.B. Dwaikat, A numerical approach for modeling the fire resistance of reinforced concrete beams, J. Cem. Concr. compos. 30 (2008) 431–443.
[9] T.T. Lie, Fire temperature–time relations, SFPE Handbook of fire protection engineering, second edition, Society of fire protection engineers, USA, 1995 Chapter 4-8.
[10] S.E. Magnusson, S. Thelandersson, Temperature-time curves of complete process of fire development, Theoretical study of wood fuel fires in enclosed spaces. Civil engineering and building series 65,Acta, Polytechnica, Scandinavia;1970.
[11] EN1991-1-2. Eurocode 1: Actions on structures. Part 1-2: general actions-actions on structures exposed to fire. Brussels: European committee for standardization; Germany, 2002.
[12] R. Feasey, A.H. Buchanan, Post-flashover fires for structural design, Fire Saf. J. 37 (1) (2002) 83–105.