A NOTE ON INCOMPLETE EXPONENTIAL FUNCTIONS

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1. INTRODUCTION

The recent paper by Chaudhry and Qadir [1] proposed the incomplete exponential functions in analogy to the incomplete gamma functions. The functions are defined by

\[ e ((x, t); \alpha) = \sum_{n=0}^{\infty} \frac{\gamma(\alpha + n, x) t^n}{\Gamma(\alpha + n)n!} \]

and

\[ E ((x, t); \alpha) = \sum_{n=0}^{\infty} \frac{\Gamma(\alpha + n, x) t^n}{\Gamma(\alpha + n)n!}, \]

where \( \gamma(\cdot, \cdot) \) and \( \Gamma(\cdot, \cdot) \) are the incomplete gamma functions defined by

\[ \gamma(a, x) = \int_{0}^{x} t^{a-1} \exp (-t) \, dt \]

and

\[ \Gamma(a, x) = \int_{x}^{\infty} t^{a-1} \exp (-t) \, dt, \]

respectively. The paper also demonstrated an application of the incomplete exponential functions to the non-central chi-square distribution.

We would like to point out that the functions given by (1) and (2) are directly related to a known function. In fact, it is easy to see that

\[ e ((x, t); \alpha) = \exp(t) \left[ 1 - Q_{\alpha} \left( \sqrt{2t}, \sqrt{2x} \right) \right] \]

and

\[ E ((x, t); \alpha) = \exp(t)Q_{\alpha} \left( \sqrt{2t}, \sqrt{2x} \right), \]

where \( Q(\cdot, \cdot) \) is the well known Marcum Q function [2] defined by

\[ Q_{M}(\gamma, \beta) = \gamma^{1-M} \int_{\beta}^{\infty} u^{M} \exp \left\{ -\frac{u^2 + \gamma^2}{2} \right\} I_{M-1}(\gamma u) \, du, \]

In the light of the relationships (3)–(4), it is worth noting that (1)–(2) provide convergent series expansions for the Marcum Q function for small values of \( t \).

The analytical as well as the computational properties of the Marcum Q function have been studied extensively, especially in the digital communication literature. See [3]–[5], Appendix A of [6], pages 394–395 and 411 of [7], [8]–[12], Chapter 4 of [13], and [14]–[16].

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Paper Received 30 April 2006; Revised 18 June 2006; Accepted a Technical Note 20 December 2006

July 2007

The Arabian Journal for Science and Engineering, Volume 32, Number 2A

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REFERENCES


