# META-COGNITION, POSITIONING AND EMOTIONS IN MATHEMATICAL ACTIVITIES

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Researchers of mathematics education have been paying attention to the affective aspect of learning mathematics for more than one decade. Different theoretical frameworks have been suggested to analyze this aspect, where we utilize in the present research the discursive framework of Evans, Morgan and Tsatsaroni. This framework enables to link students' positions to their emotions. Here, we add to this relationship the meta-cognition variable, where we study how the added variable affects students' positioning in mathematical activities. A group of three Grade 7 high-achieving students participated in the research. They used GeoGebra to learn the topic of perpendicular lines through an authentic activity. The research results indicate that most of the means of claiming leadership were metacognitive in nature and were performed to enable the advancement of the group learning of the topic of perpendicular straight lines. These means were primarily requesting actions, initiating actions, performing actions, declaring actions, taking decisions, evaluation, monitoring and regulation.

In one case, the claiming of leadership was accompanied with positive affect, namely enthusiasm, determination, and enjoyment. In the other case, the claiming of leadership was accompanied with different positive affect that included self-confidence and being proud of oneself.

Keywords: students' metacognition, students' positioning, students' emotions, mathematical activities

## INTRODUCTION

Over the years, the interest in the cognitive aspect of students' mathematical learning has broadened to the meta-cognitive aspect which is the knowledge about and regulation of one's cognitive abilities and activities in learning processes (Veenman, Van Hout-Wolters & Afflerbach, 2006). This interest, from the other side, has broadened also to the affective aspect which is believed to have mutual relationship with the cognitive aspect. The present study is related to the intersection of the metacognitive with the affective aspect of learning mathematics, when the affective aspect is represented by students' emotions related to their group positions. Specifically, the present study examines the effect of students' metacognitive processes on their positioning and related emotions, where these positioning and emotions are examined utilizing the discursive framework developed by Evans, Morgan and Tsatsaroni (2006).

### Literature review:

Researchers looked at metacognition as cognition about cognition or knowledge about knowledge (Flavell, 1976; Panaoura, Philippou & Christou, 2003; Veenman et al., 2006). Flavell (1976) was the first to use the term 'metacognition', which refers to the individual's awareness, consideration and control of his or her own cognitive processes and strategies. Since then a variety of meanings has been given to the term. Du Toit and Kotze (2009) argue that the various definitions of meta-cognitive processes in the literature, including that of Schoenfeld (1992), emphasize the monitoring and regulation of cognitive processes. In more detail, Flavell (1979) described metacognition as 'knowledge that takes as its object or regulates any aspect of any cognitive endeavor' (p. 8). Moreover, Gavelek and Raphael (1985) argue that metacognition involves promoting effective understanding through adjusting the cognitive processes involved in the activity. In addition, metacognition can affect different variables of students' learning. Panaoura, Philippou and Christou (2003) say that it coordinates cognition, affecting it and affecting students' academic success.

Researchers pointed out that metacognition is comprised of two different components connected to each other. Veenman et al. (2006) argue that the most common distinction in metacognition distinguishes between metacognitive knowledge and netacognitive skills. Flavell (1999) defines metacognitive knowledge as the knowledge or beliefs about the factors that act and interact to affect the course and outcome of cognitive enterprises. These factors include the person, the task and the strategy. The person factor concerns what a person believes about himself/herself and other people as cognitive processors. The task factor concerns the information about the object available to a person during a cognitive enterprise, where different tasks entail different mental operations. The strategy factor involves knowledge about strategies likely to be effective in achieving goals and their cognitive undertakings. On the other hand, Brown (1987) categorized metacognitive knowledge into declarative, procedural and conditional ones. Declarative knowledge refers to “knowing what”, procedural knowledge refers to “knowing how” and conditional knowledge refers to “knowing why and when”.

Metacognitive skills refer to a person's procedural knowledge for regulating one's learning activities including problem solving (Brown & DeLoache, 1978; Veenman, 2005). Moreover, these skills are implied in Flavell (1976) referring to metacognition as the active monitoring, the consequent regulation and orchestration of processes performed on cognitive objects. They are also implied in Bonds, Bonds and Peach (1992) statement that metacognition is the regulation, evaluation, and monitoring of one's thinking. They are also implied in Schraw and Dennison (1994) description of metacognition as the reflection on, understanding, and controlling of one’s learning. So, generally speaking, metacognitive skills are concerned with planning, monitoring, evaluating, orchestrating, reflecting on and controlling one's learning and cognitive processes.

To try to describe how and when metacognitive regulation works, Kluwe (1982) uses the term ‘executive processes’ to denote both monitoring and regulating strategies. Executive monitoring processes are processes one takes in order to decide how (a) to identify the task on which one is currently working, (b) to check on current progress of that task, (c) to evaluate that progress, and (d) to predict what the outcome of that progress will be. Moreover, executive regulation processes are those that are “directed at the regulation of the course of one’s own thinking” (p.212). Furthermore, these processes can involve one’s decisions regarding (a) allocation of resources to the current task, (b) determining the order of steps to complete the task, and (c) setting the intensity or the speed at which one should work on the task (Hacker, 1997; Papaleontiou-Louca, 2008). A third attempt to describe how metacognition works is that of Nelson and Narens (1990) who suggest that learners do that by monitoring and controlling their cognitive processes. In the process of control, the meta-level modifies the object-level, so either it changes the state of the object-level process or changes the object-level process itself. The result of the control process could be (1) initiating an action, (2) continuing an action, or (3) terminating an action. Nelson and Naren (1990) argue that a monitoring component is needed that is logically (even if not always psychologically) independent of the control component (p. 127). Moreover, the main methodological tool for generating data about metacognitive monitoring consists of "the person's subjective reports about his or her introspections" (p. 127).

In addition, researchers suggested ways to encourage students to use metacognitive processes (e.g., Spiller & Ferguson, 2011). Flavell (1979) emphasizes that metacognition improves with practice. Schoenfeld (1992) describes ways that students can practice to monitor and evaluate their performance on math problems. For example, pause frequently during problem solving to ask themselves questions such as “What am I doing right now?” Spiller and Ferguson (2011) say that if we want students to use metacognitive processes, we need to encourage them to consider the nature and sequence of their own thinking processes. Chauhan and Singh (2014) say that as students become more skilled at using metacognitive strategies, they become confident and more independent as learners. This independence leads to ownership as students realize their ability to answer and pursue their own academic needs.

**Metacognition in learning mathematics:**

Metacognition has attracted the attention of mathematics education researchers. Schoenfeld (1992), as described above, suggests ways that students perform to use metacognition in mathematical problem solving. Barbacena and Sy (2015) examined university students' use of metacognitive skills in problem solving and found that the students exhibited metacognitive awareness, metacognitive evaluation and metacognitive regulation that operated as pathways from one to another metacognitive function (metacognitive awareness, metacognitive regulation and metacognitive evaluation). Moreover, Awawdeh-Shahbari, Daher and Rasslan (2014) investigated the relationship between mathematical knowledge and cognitive and metacognitive processes exhibited by students from Grades 6, 7, and 8 who engaged in a set of model-eliciting activities in groups of 4-5 students each. The results of the study showed that the highest percent of cognitive processes and lowest percent of metacognitive processes occurred amongst the Grade 6 students, while the lowest percent of cognitive processes and highest percent of metacognitive processes occurred amongst the Grade 8 students. The Grade 6 students' metacognitive processes were more awareness than regulation and evaluation skills. Conversely, the Grade 7 and 8 students employed more regulation and evaluation processes.

Panaoura et al. (2003) built a questionnaire for the assessment of metacognition in mathematics learning appropriate for young children. Using the questionnaire, they found that pupils had a very poor knowledge about their cognitive abilities while attempting to solve a non-routine problem. Moreover, Panaoura and Panaoura (2006) found that processing efficiency had a coordinator role on the growth of mathematical performance, while self-image, as a specific metacognitive ability, depended mainly on the previous working memory ability. In the present research, we examine the effect of meta-cognition on students' affect represented in their emotions and mediated by their positioning.

**Emotions in mathematics education:**

Hannula (2004) says that emotion could be regarded as the most fundamental concept in affect. Grootenboer and Marshman (2016) describe emotions as affective responses to a particular situation that are temporary and unstable. Hannula (2004) says that though there is no agreement among researchers regarding what emotions are, they agree that emotions satisfy certain conditions. First, they are seen in connection with personal goals. Second, they involve physiological reactions. Third, emotions are functional, having an important role in human adaptation.

Martínez-Sierra (2015) claims that most of the research on students' emotions in mathematics education focuses on their role in mathematical problem solving. Such studies have confirmed that people tend to experience similar emotions in the process of problem-solving. At the same time, they experience different emotions in the same mathematical activity. For example, Op' T Eynde, De Corte and Verschaffel (2007) found that students experience different emotions during the solution of a mathematical problem. These emotions can be annoyance, frustration, angriness, worry, anxiousness, relieve, happiness or nervousness. Martínez-Sierra (2015) found that students experienced satisfaction when being able to solve a problem. They felt boring when not understanding the teacher's explanation. In the present research, we associate students' emotions with their positions in the group of learning.

**Students' positioning and emotions in mathematics learning:**

The study of the link between students' positioning and emotions has been going on more than one decade now (see Morgan, Evans, & Tsatsaroni, 2002), but the discursive framework suggested as an analyzing tool of this link has not been utilized by researchers for studying the affective aspect of students' learning.This framework draws on social semiotics, pedagogic discourse theory and psychoanalysis, and studies emotion as discursive positioning. Doing so, it takes into consideration positions available to the mathematics learners through their learning practices. These positions enable and constrain the learners' emotions, which makes learners' emotions to be considered as shaped by power relations.

The discursive analysis of students' emotions and positioning has two phases: the structural and the textual. In the structural phase, learners' positionings are analyzed. Evans, Morgan and Tsatsaroni, in their writings about discursive analysis describe the positionings taken care of in the structural analysis: Helper and seeker of help (helper positioned more powerfully), collaborator and solitary worker, director of activity and follower of directions (the latter less powerful), evaluator and evaluated, insider and outsider.The second phase of discursive analysis (the textual analysis) has two functions (Evans, 2006): (a) showing how positionings in social interactions are actually taken up by the participants, and (b) providing indicators of emotional experience. Furthermore, in the textual analysis, indicators of interpersonal relationship and emotional experience are considered.

Little research has utilized the discursive framework of Evans, Morgan and Tsatsaroni. This includes research done by the founders, as well as few additional researches such as Daher (2015) and Daher, Swidan and Shahbari (2015). The little research that uses the framework means that the potentialities of the framework as an analysis method that takes into consideration two aspects of learning are not utilized fully. It is our intention to follow the few researches described above and utilize the discursive framework further, when adding to the social and emotional aspects a third aspect; that of metacognition.

**Research rationale and goals:**

In the present research we intend to study the interaction between students' meta-cognitive processes from one side, and their positionings and related emotions from the other side. Thus the present research attempts to shed light on this interaction that, as mentioned above, has not been attended to previously or little attended to. We attempt to do that, specifically, when a group of seventh grade students works with Geogebra to solve an authentic mathematical problem related to the relationship between the parameters of two linear functions that represent perpendicular straight lines. The discursive analysis was used to analyze students' positioning and emotions in their exploration work.

**Research question:**

How meta-cognitive processes affect students' positioning and emotions during solving authentic mathematical problems?

## RESEARCH METHODOLOGY

**Research setting and participants:**

A group of seventh grade high-achieving students participated in the research: Kamar, Salim and Ranin (pseudonyms). We chose high-achieving students, for we expected that they would use metacognitive processes more than other students. This expectation follows Cera, Mancini and Antonietti (2013) who found that the acquisition of metacognitive knowledge, skills and attitude are linked to autonomy in study and to self-efficacy. These later variables could be possessed by high-achieving students more than other students.

The three participating students did not work with GeoGebra before, and they were introduced to it in two hours' time. This introduction included the introduction to its various drawing tools. Furthermore, the students were introduced to the topic of perpendicular lines for the first time. This happened through an authentic activity in which they took pictures of perpendicular lines in everyday life phenomena. The authentic activity text requested the group to insert one of the pictures into GeoGebra interface and explore the relationship between the parameters of perpendicular straight lines in the picture.

**Data collection:**

We used two collecting tools in the present research to collect data about the participating students' metacognitive processes, their claimed positions in the group and their related emotions. These tools were videoing and interviews.

We videoed the learning of the group of students with two cameras, and at the end of every activity, we interviewed the students individually about the three research aspects. Examples on the interview questions are: What emotions did you have during the activity? Why did you have such emotions (for example, why were you satisfied?).

**Data analysis**

We transcribed the videos and interviews putting emphasis on indicators of meta-cognition (for example sentences as "we should measure the angle to be sure"), positioning (for example putting a hand on the face, as if annoyed), and emotions (for example leaning forward in tension or backward to relax). As described above, we utilized the discursive framework developed by Evans, Morgan and Tsatsaroni (2006) for analyzing the participating students' positioning and emotions. In addition, we depended on Flavell (1976) to analyze the participating students' metacognitive processes during mathematical learning. Flavell (1976), as described above, refers to metacognition as the active monitoring, the consequent regulation and orchestration of processes performed on cognitive objects.

In more detail, indicators of claiming leadership are processes that make the person advance the learning of the group, or influence this learning. Indicators of claiming the collaborator position are processes that show a person as working together with the other group members.

**The learning material:**

The students worked on an authentic activity, where they were requested to take pictures of perpendicular lines in the nature or everyday life, put the pictures in GeoGebra interface and find the relation between the parameters of these perpendicular lines.

The activity text:

a) Take pictures of perpendicular lines in real life or nature.

b) Insert the pictures into the interface of GeoGebra.

c) Draw lines in GeoGebra that fit the perpendicular lines in the picture.

d) Investigate the relationship between the slopes of the two perpendicular lines.

**The technological tool:**

The technological tool utilized by the group of students was GeoGebra. This tool is free and available in many languages. It enables the investigation of mathematical concepts and relations in different mathematical domains, especially geometry and Algebra, but not only. The tool has different fields that enable working with mathematical objects and relations; especially writing algebraic rules or formulae, drawing functions or geometric shapes, observing formulae of already worked-with objects, and manipulating mathematical objects.

In the present research, the students utilized GeoGebra to work on specific components of real life objects. GeoGebra enables the insertion of images in its interface, drawing geometric shapes on the images and measuring specific features of these shapes, as the length of lines. The students drew straight lines that fit perpendicular lines in the real life images, measured angles between them and manipulated them in order to make the angle between them a right angle.

## RESULTS

The results report students' meta-cognitive processes and how they affected their positioning and related emotions while utilizing GeoGebra for solving authentic problems related to the perpendicular lines topic. We report the events chronologically. The events are named after the metacognitive processes utilized in order to claim group positions. Emotions are described as part of the events and consequences of the positions claimed by the students.

### Managing the group's learning as means to claim leadership

The students were requested to look for a couple of perpendicular straight lines in one of the real life pictures that they collected, to insert the picture in GeoGebra, and to draw two straight lines that fit the couple of perpendicular lines in the picture. The students chose one of the pictures that included different couples of perpendicular straight lines and put it in GeoGebra. Kamar inquired about the couple of lines they should first consider. Ranin suggested taking the two straight lines that constitute the right upper corner of the paper in the picture (two edges of the paper).

**Picture1 : drawing two perpendicular lines in GeoGebra**

| | | |
|---|---|---|
| 37 | Kamar | Which picture to take? |
| 38 | Salim | The paper on the table |
| 39 | Ranin | This one. |
| | | [she pointed at the paper in the picture] |
| | | The paper. |
| 40 | Kamar | Let's put the picture in GeoGebra |
| | | [She goes to GeoGebra and inserts in its interface the agreed picture] |
| | | These lines are intersecting, no, no, I mean perpendicular. |
| | | [She points at the lines in the picture that she inserted in the interface of GeoGebra, saying with determination] |
| | | We need to take care of the two edges of the paper. |
| | | [She enthusiastically worked with GeoGebra, determined to lead the group in the solution of the mathematical problem] |
| 41 | Ranin | Yeh, the paper. |
| | | [The three of them pointed at the two lines that constituted the right upper corner of the paper] |
| 42 | Ranin | Yes here. |
| | | [Kamar and Ranin pointed with their finger at the two perpendicular lines, while Ranin passed her finger at them] |

**Transcript 1: Choosing a pair of perpendicular lines and putting them in GeoGebra**

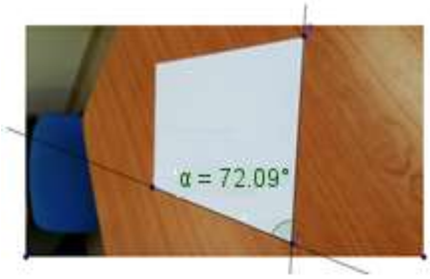
Kamar seems to be the active leader of the group. She claimed her leadership by different means. First, by means of requesting actions, in our case requesting that the group decides which picture to take (R37). Second by means of initiating actions, in our case initiating the action of putting the chosen picture in GeoGebra interface (R40). Third, by means of performing actions, in our case inserting the picture in GeoGebra interface (R40). Fourth, by means of declaring actions, in our case declaring the need to take care of the two lines at the corner of the paper (R40).These means of claiming leadership are metacognitive in nature and were performed to enable the advancement of the group learning, in our case of the topic of perpendicular straight lines.

In addition to the said above, Kamar, being the leader of the group, led the group enthusiastically and with determination (R40). In the interview, Kamar continued to talk with enthusiasm about her position as leading the advancement of the group learning. She said: "I knew what we should do. It was important for me to help the group go forward and solve the problem".

**Claiming leadership by evaluating, requesting justification of claims andmonitoringthe group's work**

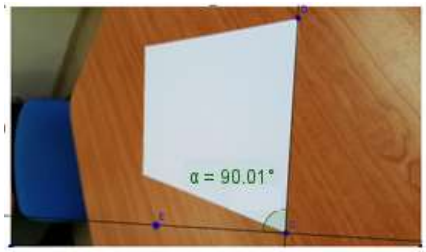
The students turned to fit two straight lines to the two edges of thepaper. Kamar started this task by assigning two points on the bottom edge of the paper and then drew a straight line through these two points (R43).Then she did the same thing to the right edge (R45). After drawing the two lines that fitted the two edges of the paper, the students expected the two straight lines to be perpendicular, but they wanted to verify this issue, so they decided to measure the angle between them (R47-R50). The angle turned to be not a right angle, which made the group members disappointed. The first one to talk was Ranin (usually a collaborator) who seemed frustrated (R51). Salim (a collaborator) shook his head quietly, agreeing with Ranin. Kamar (the leader) seemed anxious having a slight smile. She got quiet for few seconds, and then she gave directions what to do to make the two straight lines perpendicular (R53). She got closer to GeoGebra interface and started to change the slope of one of the straight lines by dragging it. Doing so, she was observing the angle between the two straight lines. She succeeded to make the angle approximately a right one. This changed the negative emotions of the group and turned them into positive ones.

| | | |
|---|---|---|
| 43 | Kamar | Is it O.K. like this? |
| | | [Kamar assigned two points on the bottom edge of the paper, and drew a straight line through them] |
| 44 | Salim | Yeh |
| 45 | Salim and Kamar (together) | |
| | | Let's now draw the other straight line. |
| | | [Kamar assigned two points on the right edge of the paper and drew a straight line through them] (See Picture 1). |
| | | [Kamar seemed enjoying working with GeoGebra as a tool for drawing the straight lines] |
| 46 | Ranin | Right |
| | | [Ranin nodded with her head, talking in a low voice] |
| 47 | Salim | We are almost there [..5..] but is it really 90 degrees? |
| 48 | Kamar | Salim asked whether it is 90 degrees. We should verify that. |
| 49 | Salim | Verify. |
| | | [He was talking with self-confidence] |
| 50 | Kamar | Kamar measured the angle, but it turned to be not a right angle (See Picture 2). |
| | | [The three of them leaned towards GeoGebra interface. Ranin seemed to be frustrated because the angle was not a right angle] |




**Picture2: Measuring the angle showed it not to be a right one**

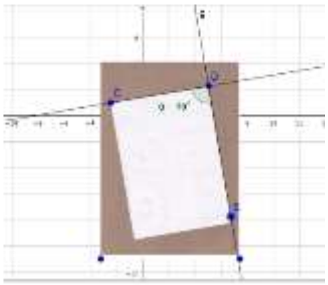
| | | |
|---|---|---|
| 51 | Ranin | No, not a right angle. |
| | | [She seemed not pleased with the fact] |
| 52 | Salim | [Quietly] Not exactly |
| 53 | Kamar | [Kamar seemed anxious but had a slight smile] [..4..] |
| | | We should move the slope a little. |
| | | [She moved the bottom line counterclockwise until it became very close to 90] (See Picture 3) |

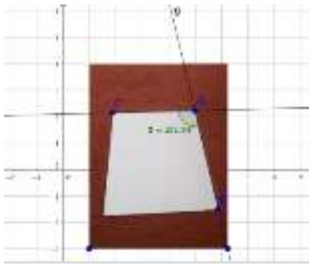



**Picture3: changing the slope of the bottom line to turn the angle into a right angle**

| | | |
|---|---|---|
| 54 | Salim | We got a right angle. |
| 55 | Ranin | Great |
| | | [Ranin smiled and seemed happy with the result] |
| 56 | Teacher | Fine, but we need to think why the original angle in the picture did not turn to be a right angle. |
| 57 | Ranin | Right. It is strange. |
| 58 | Kamar | [Takes the cellular and goes to take a picture for an A4 paper] |
| | | I think it is the photography angle. |
| 59 | Salim | What do you mean? |
| 60 | Kamar | |

[Takes pictures of the A4 paper from various angles, and puts them in GeoGebra to arrive at different angles] (See Picture 4)





**Picture 2: Inserting pictures of A4 papers taken from different photography angles**

| | | |
|---|---|---|
| 61 | Ranin | Fantastic. |
| 62 | Salim | We should be careful of the photography angles when we want to measure angles. |
| 63 | Kamar | All is understood now. Let us continue.<br>[she seemed content] |

**Transcript 2: Verifying the angle between two lines expected to be perpendicular**

Here, Kamar and Salim seem to be the leaders of the activity. Kamar continued to claim leadership through doing (R43, R45, R50, R53 and R60), initiation (R45) and taking decisions (R53, R58 and R63). On the other hand Salim claimed leadership through evaluation (R44, R47, R52, R54 and R62) and monitoring (R47, R49). In the interview, Salim expressed the need for continuous evaluation of the actions that the group performs to solve the problem. Salim said: "I always go back and see if anything went wrong, even if it seems that nothing went wrong".

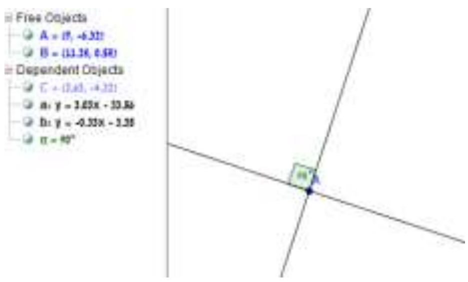
Leading, through working with GeoGebra, made Kamar enjoy her positioning as a leader of the group's learning (R45). She also was content arriving at the reasons behind their mathematical doing [R63]. At the same time, Salim's monitoring (for example to examine whether their claim about the measure of the angle was correct) made him confident of himself as a leader of the group (R49). Furthermore, Ranin, being insider, moved between negative emotions - when the result of their work was not as expected and positive emotions- when the result of their work agreed with their goal (R51, R55). In the interview, Ranin said: "Reaching the goal of the activity is marvellous. It makes me glad and want to learn more and more".

**Claiming leadership by monitoring and regulation**

Kamar led the group again through the exploration of the mathematical phenomenon which is the relationship between two perpendicular lines (R64), stating the requirement of the question. The teacher helped them technically how to find the slopes of the lines using GeoGebra (R67). Kamar, after adjusting the lines equations according to the form $y=mx+b$, stated one property of the perpendicular lines, where this property was related to their slopes' values. The teacher urged them to find another property (R73). Salim suggested working with other lines to make the exploration of the phenomenon easier; namely lines that one of them has an integer slope (R74). This suggestion led the students to discover the required relationship (R75-R82).

| | | |
|---|---|---|
| 64 | Kamar | What should we do now?<br>[She reads in a low voice the question, holding a pen] |

| | | |
|---|---|---|
| | | The question requests to find the relationship between the slopes of the perpendicular lines. |
| | | [She shakes the pen then puts it on her cheek, and looks at GeoGebra interface. All the time seems to be thinking] |
| 65 | Salim | Yeh, the slopes of lines. |
| 66 | Kamar | Let's find the slopes of the lines. |
| 67 | Teacher | GeoGebra gives the slope of a line by clicking the right side of the mouse and choosing the for y=mx+b. |
| 68 | Kamar | [Adjusts the two equations of the lines into y=mx+b, then points at each of the resulting slopes, touching the screen] |
| | | This slope is positive, and this is negative. |
| 69 | Salim | Right. This one is an increasing function, and this one is a decreasing function. |
| | | [He too points at the slopes but without touching them] |
| 70 | Ranin | One is positive and one is negative. |
| 71 | Kamar | So, what do we conclude? |
| 72 | Ranin | They are different. |
| 73 | Teacher | What else can you say about their relationship? |
| 74 | Salim | [..5..] |
| | | Else! |
| | | Why not working with other lines that have integer slopes? |
| | | [He seemed proud to suggest a new way to explore the slopes of the perpendicular lines] |
| 75 | Kamar | Kamar did not seem pleased with Salim's suggestion. Nevertheless, she said with hesitation] |
| | | Right, what happens then? |
| 76 | Salim | Draw a line and draw a perpendicular line that intersects it. |
| 77 | Kamar | [Draws a line between two points, then draws a perpendicular line to it] |
| | | Now the slopes. |
| | | [She changes the equations of the line into the slope equations. Then she drags the first line to be drawn till its slope became approximately an integer] (See Picture 4). |




**Picture 4: two perpendicular lines, where one of the slopes is almost an integer number**

| | | |
|---|---|---|
| 78 | Salim | Fantastic!! One of the slopes is three and the other is one third. |
| 79 | Ranin | One is the inverse of the other. |
| 80 | Salim | Not the inverse. Their product is minus one not one. |
| 81 | Kamar | Right. |
| 82 | Ranin | Aha, the product is minus 1, excellent. |
| | | [She seemed very happy with the result] |
| 83 | Salim | What about other perpendicular lines? |

**Transcript3: investigating the characteristics of the perpendicular lines**

In spite of Kamar's attempt to claim leadership, Salim seemed to succeed in claiming it more than Kamar at this phase. He did that mainly by carrying out the same meta-cognitive processes as he did before. These meta-cognitive processes were: Requesting action (R76), evaluating (R80), monitoring (R83) and regulating (R74). At the same time, he exhibited knowledge (R78).

Leading, through using meta-cognitive processes, Salim seemed proud of himself (e.g., R74). At the same time Salim's behaviour resulted in Kamar's having negative emotions (R75), as if feeling she was not still a leader of the group's learning. In the interview, Salim said: "It was important to suggest a new way for performing the activity. I felt proud for that. You always have to stop and ask if there is a better way to solve the problem at hand".

## DISCUSSION AND CONCLUSIONS

The present study intended to examine the effect of meta-cognitive processes on students' claiming of positioning and their related emotions. The research results indicate that most of the means of claiming leadership were metacognitive in nature and were performed to enable the advancement of the group learning of the topic of perpendicular straight lines.These processes were requesting actions, initiating actions, performing actions, declaring actions, taking decisions, evaluation, monitoring and regulation. The previous processes are metacognitive except 'performing actions' which is a behavioral process. This link of the ability to perform metacognitive processes to claiming leadership is described in Black, Soto and Spurlin (2016) who say that metacognitive ability is one of the main components of the 'leader developmental readiness'. This is also supported by Hannah and Avolio(2010) who say that leaders’ ability to develop is promoted through self-awareness, self-complexity, and meta-cognitive ability.

As metacognitive processes were associated with the group members' claim of leadership, group leadership was associated with the members' experiencing of emotions. In addition to the said above, different group leaders experienced sometimes similar emotions, but in other times, they experienced different emotions. For example, in one case – the case of Kamar, the claiming of leadership was accompanied with positive affect, namely enthusiasm, determination, and enjoyment. In the other case – the case of Salim, the claiming of leadership was accompanied with positive affect that differed from that of Kamar. In this case, the positive affect was self-confidence and being proud of oneself. Salim's experiencing of emotions were affected by the stage in which he claimed the group's leadership, where he claimed leader at a later stage after Kamar claimed this leadership first. To challenge the leadership of another and claim it would result in self-confidence and pride as Salim did experience.

Furthermore, Ranin, being insider, moved between negative affect resulting from unexpected mathematical results and positive affect resulting from arriving at results that agreed with the learning goals. Sometimes, the positive affect of one member was accompanied with the negative affect of the other, as when the feeling proud of oneself of Salim was accompanied with the uneasiness of Kamar. This contrast of affect between two group members is a result of the two members challenging each other to claim the group leadership.

In addition to the said above,researchers claim that positive affect, on the one hand,eases the person's effort exertion and, on the other, increases interest and feeling of liking, thus supporting futureengagement with the same or similar tasks (Efklides & Petkaki, 2005). Thus, we should try to treat the contrast of affect described above by educating students that it is normal to claim leadership part of the lesson, and, allow other students to lead in other parts, for learning is a group and mutual effort and not a one man effort.

In light of the results and discussion presented above, positioning could be regarded as mediating between the use of metacognitive processes and emotions. These findings add to the existing literature that students' perceived self-regulation is significantly positively correlated with positive emotions (Boekaerts, Pintrich, & Zeidner, 2000; Carver & Scheier, 1990). Other findings in this field is that of Pekrun, Götz, Titz and Perry (2002) who found that attention, self-regulation and motivation mediate the relationship between learning and achievement on one hand and emotions of the other hand. The present research could be regarded a step forward towards understanding the relationship between metacognition and emotions. Future research is needed to verify the relationship between the various constructs of students' learning: cognition, metacognition, positioning and emotions.

## REFERENCES

Awawdeh-Shahbari, J., Daher, W. & Raslan, S. (2014). Mathematical knowledge and the cognitive and metacognitive processes emerged in model-eliciting activities. *International Journal of New Trends in Education and Their Implications, 5* (2), 209-2019.

Barbacena, L., &Sy, N. (2015). Metacognitive Model in Mathematical Problem Solving. *Intersection, 12*(1), 16-22.

Black, H., Soto, L., &Spurlin, S. (2016). Thinking about thinking about leadership: Metacognitive ability and leader developmental readiness. *New Directions for Student Leadership*, 149, 85-95.

Boekaerts, M., Pintrich, P. R., &Zeidner, M. (2000). *Handbook of self-regulation*. San Diego, CA, US: Academic Press.

Bonds, C. W., Bonds, L. G., & Peach, W. (1992). Metacognition: Developing independence in learning. *The Clearing House, 66*(1), 56-59.

Brown, A. (1987). Metacognition, executive control, self-Regulation and other more mysterious mechanisms. In F.F. Weinert& R.H. Kluwe (Eds.), *Metacognition, motivation and understanding* (pp. 65-116). New Jersey: Lawrence Erlbaum Associates.

Brown, A. L., &DeLoache, J. S. (1978). Skills, plans and selfregulation. In R. S. Siegler (Ed.), *Children's thinking: What develops* (pp. 3-35). Hillsdale, NJ: Erlbaum.

Carver, C. S., &Scheier, M. (1990). Principles of self-regulation: Action and emotion. In E. T. Higgins & R. M. Sorrentino (Eds.), *Handbook of motivation and cognition: Foundations of social behavior* (Vol. II, pp. 3−52). New York: Guilford Press.

Cera, R., Mancini, M., & Antonietti, A. (2013). Relationships between metacognition, selfefficacy and self-regulation in learning. *Journal of Educational, Cultural and Psychological Studies, 7*, 115 – 141.

Daher, W. (2015). Discursive Positionings and Emotions in Modelling Activities. *International Journal of Mathematical Education in Science and Technology*, 46(8), 1149-1164.

Daher, W., Swidan, O., & Shahbari, J. (2015). Discursive positionings and emotions in a small group's learning of geometric definitions. In K. Krainer&N.Vondrov´a (eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education (CERME 9)* (pp.1160-1166). Prague, Czech Republic: ERME.

Du Toit, S. &Kotze, G. (2009). Metacognitive strategies in the teaching and learning of mathematics. *Pythagoras, 70*, 57–67.

Efklides, A., &Petkaki, C. (2005). Effects of mood on students' metacognitive experiences. *Learning and Instruction*, *15*, 415–431.

Evans, J. (2006). Affect and emotion in mathematical thinking and learning. In J. Maasz& W. Schlöglmann, *New mathematics education research and practice* (pp. 233–256). Rotterdam: Sense Publishers.

Evans, J., Morgan, C., & Tsatsaroni, A. (2006). Discursive positioning and emotion in school mathematics practices. *Educational Studies in Mathematics: Affect in Mathematics Education: Exploring Theoretical Frameworks, A PME Special Issue, 63*(2), 209–226.

Flavell, J. (1976). Metacognitive Aspects of Problem Solving. in L. Resnick, (Ed), *The Nature o/Intelligence* (Pp. 231-235). Erlbaum Associates: New Jersey.

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive–developmental inquiry. *American Psychologist, 34*, 906–911.

Flavell, J. (1987). Speculations about the nature and development of metacognition. In F. Weinert& R. Kluwe (Eds), *Metacognition, Motivation and Understanding* (21-29). London: LEA.

Flavell, J. (1999). Cognitive development: children's knowledge about the mind. *Annual review of psychology, 50*, 21-45.

Gavelek, J. R., Raphael, T. E. (1985), Metacognition, instruction, and questioning, In D. L. Forrest-Pressley, G. E. MacKinnon, T.G. Waller (Eds.), *Metacognition, cognition, and human performance* (Vol. II, pp. 103-132), Orlando: Academic Press.

Grootenboer, P., &Marshman, M. (2016). *Mathematics, affect and learning: Middle school students' beliefs and attitudes about mathematics education*. Singapore: Springer.

Hannah, S.T., &Avolio, B.J. (2010). Ready or not: How do we accelerate the developmental readiness of leaders? *Journal of Organizational Behavior, 31*, 1181-1187.

Hannula, M. (2004). Affect in mathematics education - exploring theoretical frameworks. In M. J. Hoines, & A. B. Fuglestad (eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol I, pp. 107–136). Bergen, Norway: International Group for the Psychology of Mathematics.

Gustavo Mart´ınez-Sierra. Students' emotional experiences in high school mathematics classroom. In K. Krainer&N.Vondrov´a (eds.), Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education (CERME 9) (pp.1181-1187). Prague, Czech Republic: ERME.

Kluwe, R. H. (1982). Cognitive knowledge and executive control: metacognition. In D. R. Griffin (ed.), *Animal Mind - human Mind* (pp.201-224). New York: Springer-Verlag.

Morgan, C., Evans, J., & Tsatsaroni, A. (2002), Emotion in school mathematics practices: A contribution from discursive perspectives, In P. Valero & O. Skovsmose (Eds.), *Proceedings of the Third International Conference Mathematics Education and Society* (pp. 400–413), Helsingor, Denmark: Centre for Research in Learning Mathematics, The Danish University of Education.

Nelson, T. O., &Narens, L. (1990). Metamemory: A theoretical framework and new findings. In G. H. Bower (Ed.), *The psychology of learning and motivation* (pp. 125-173). New York: Academic Press.

Op' T Eynde, P., De Corte, E., &Verschaffel, L. (2007). Students' emotions: A key component of self-regulated learning? In P. A. Schutz & R. Pekrun (Eds.), *Emotion in education* (pp. 185–204). Burlington, MA: Academic Press.

Panaoura, A., &Panaoura, P. (2006), Cognitive and metacocognitive performance on mathematics, In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings 30th conference of the international group for the psychology of mathematics education*, vol. 4, (pp. 313-320), Prague, Czech Republic: PME.

Panaoura, A., Philippou, G., & Christou,C. (2003). *Young pupils' metacognitive ability in mathematics*. Paper presented at the Third Conference of the European Society for Research in Mathematics Education.

Papaleontiou-Louca, E. (2008). *Metacognition and theory of mind*. Newcastle: Cambridge Scholars Pub.

Pekrun, R., Götz, T., Titz, W., & Perry, R. P. (2002). Academic emotions in students' self-regulated learning and achievement: A program of qualitative and quantitative research. *Educational Psychologist, 37*(2), 91−105.

Schoenfeld, H. (1992). Learning to think mathematically: problem solving, metacognition and sense making in mathematics. In D. A. Grouws (Ed), *Handbook of research on mathematics teaching and learning* (pp. 334-368). New York: McMillan.

Schraw, G. & Dennison, R. S. (1994).Assessing metacognitive awareness. *Contemporary Educational Psychology*, 19, 460-475.

Spiller, D., & Ferguson, P.B. (2011). *Teaching strategies to promote the development of students' learning skills*. Hamilton, New Zealand: Teaching Development Unit.

Veenman, M. V. J. (2005). The assessment of metacognitive skills: What can be learned from multimethod designs? In C. Artelt, & B. Moschner (Eds), *Lernstrategien und metakognition: Implikationen fur forschung und praxis* (pp. 75–97). Berlin: Waxmann.

Veenman, M.V.J., Van Hout-Wolters, B.H.A.M, &Afflerbach, P. (2006). Metacognition and Learning: Conceptual and Methodological Considerations. *Metacognition and Learning, 1*, 3-14.